Overview of eRHIC design

- What is eRHIC
- Evolution of eRHIC concept
- Recent developments
- Main elements of the concept with FFAG arcs
- Solutions we found
- Questions remaining

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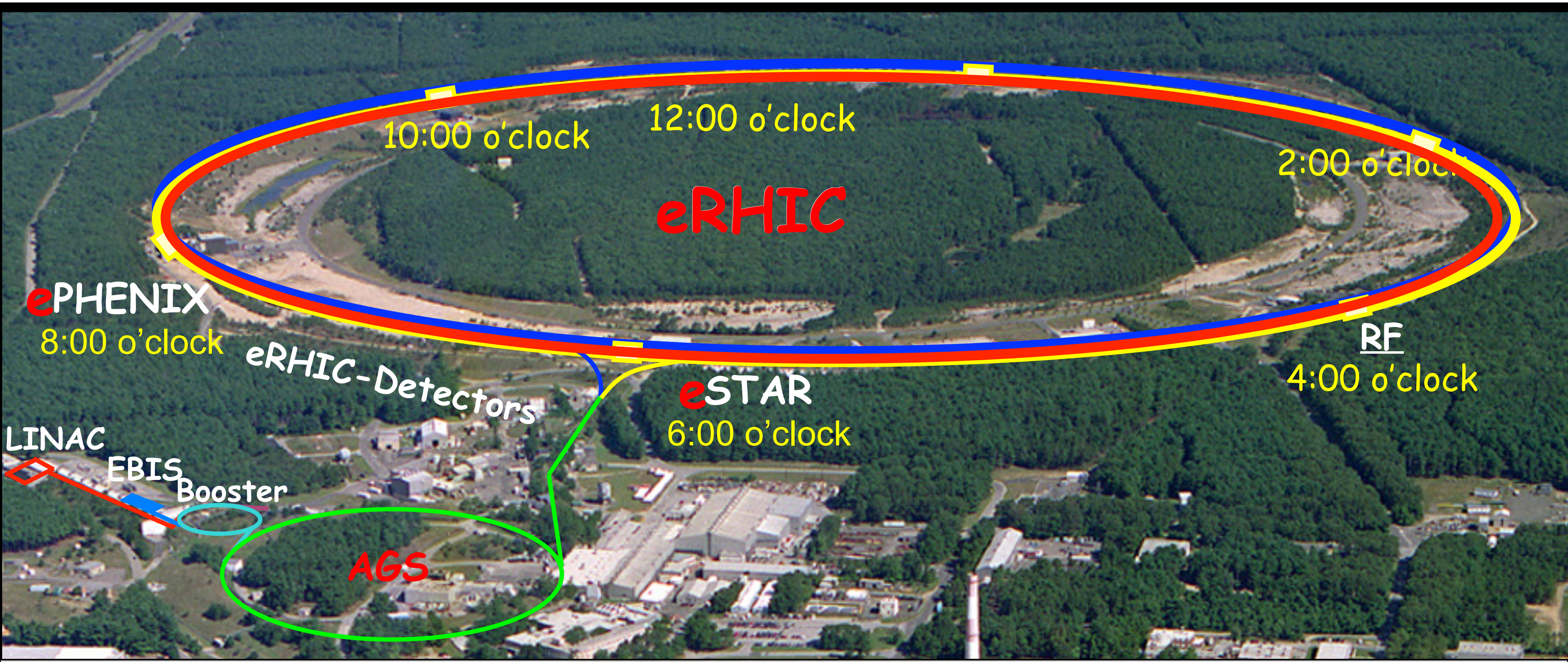
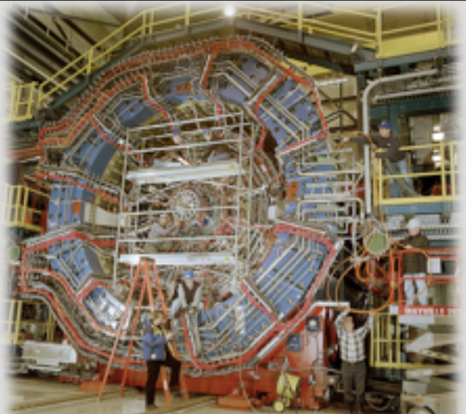
From RHIC to eRHIC





eRHIC: QCD Facility at BNL

ERL

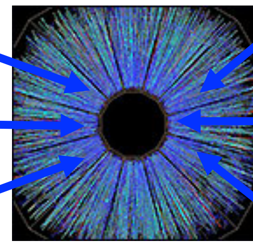
RHIC

Un-polarized and polarized leptons 10-30 GeV

e^-

e^-

e^+



p

Polarized protons 50-250 GeV

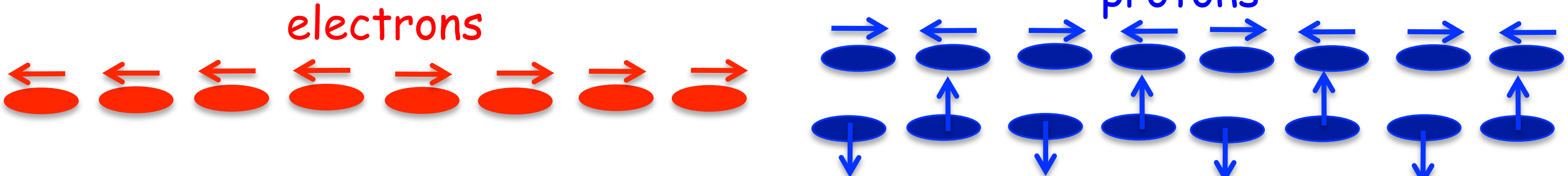
Light ions (d,Si,Cu)
Heavy ions (Au,U)
50-100 GeV/u

Polarized light ions
He^3 166 GeV/u

80% e^- beam polarization

Polarized positrons only if needed

Center mass energy range: $\sqrt{s}$=30-175 GeV;
Luminosity ~ 10^{33}-10^{34} $cm^{-2}\,sec^{-1}$



Introduction

- We had completed staged approach for eRHIC based on 6-pass 5-to-30 GeV ERL
- This eRHIC design was reviewed by external review committee in August 2011
  - We followed up on the committee suggestions
- We performed bottoms-up engineering cost estimate for eRHIC with 5 GeV ERL with optimum at ~ $550M
- We established a well defined R&D program including:
  - Coherent Electron Cooling PoP & linac-ring beam-beam effects
  - Polarized Gatling gun
  - R&D ERL
  - Magnet prototyping
  - FFAG lattice

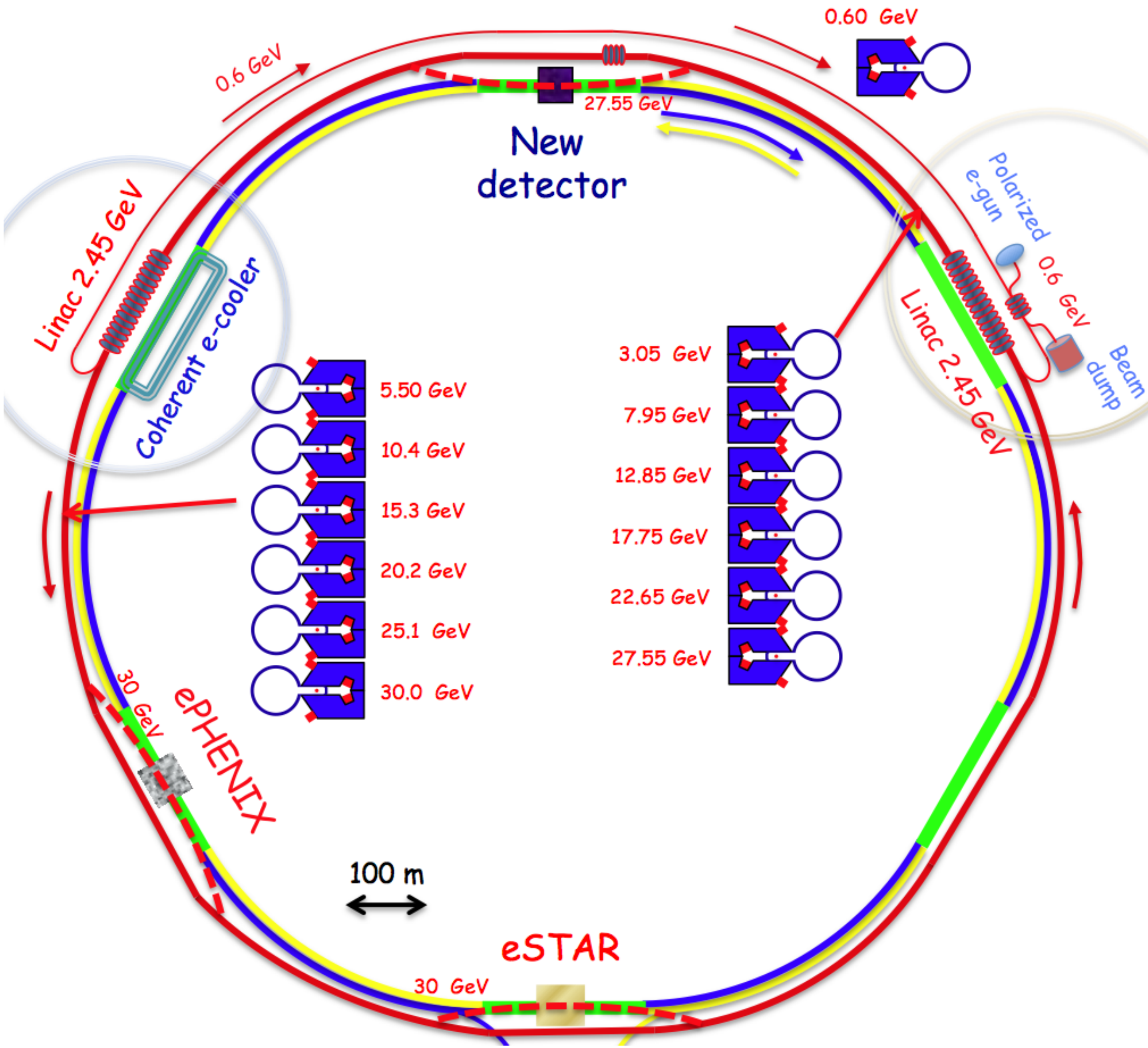
- Starting about an year ago eRHIC group pursues a very serious challenge: to fit eRHIC with 10 GeV ERL and luminosity at 10^{33} cm^{-2} sec^{-1} into $0.5B
- Our previous cost estimate indicated that the most economic option would be reducing number of ERL arcs and increasing number of passes through linacs: e.g. an FFAG
  - We (*read D. Trbojevic*) pursued FFAG option for eRHIC ERL arcs for about ten years from the onset of eRHIC
- While we solved many key issues for such design, a large number of questions and effects to study remains
- **Hence, we present you an up-to-date SNAP-SHOT of our design**
- We will try to indicate clearly what beam dynamics and technical issues are solved or what are to-be-solved

Main elements of the previous concept

- Use ERL for electrons to reach high luminosity at high energy
- Use up to 6 passes through main ERL linac(s) to reach 5 to 30 GeV
- Use up to 6 individual magnet strings in RHIC tunnel to transport beams
- Cool hadron beam 10-fold in all directions using coherent electron cooling (CeC)
- Use small magnets with gaps of 5 mm (and 10 mm at two lowest orbits)
- Use harmonic jump to collide electrons with 50 GeV and 250 GeV hadrons
- Use recent advances in SC quadrupole technology to design IR with β^* to 5 cm
- Use crab-crossing to avoid IR SR background
- Develop Gatling polarized electron gun with 50 mA CW current
- These assumptions bring eRHIC top luminosity to 10^{34} cm^{-2} sec^{-1}

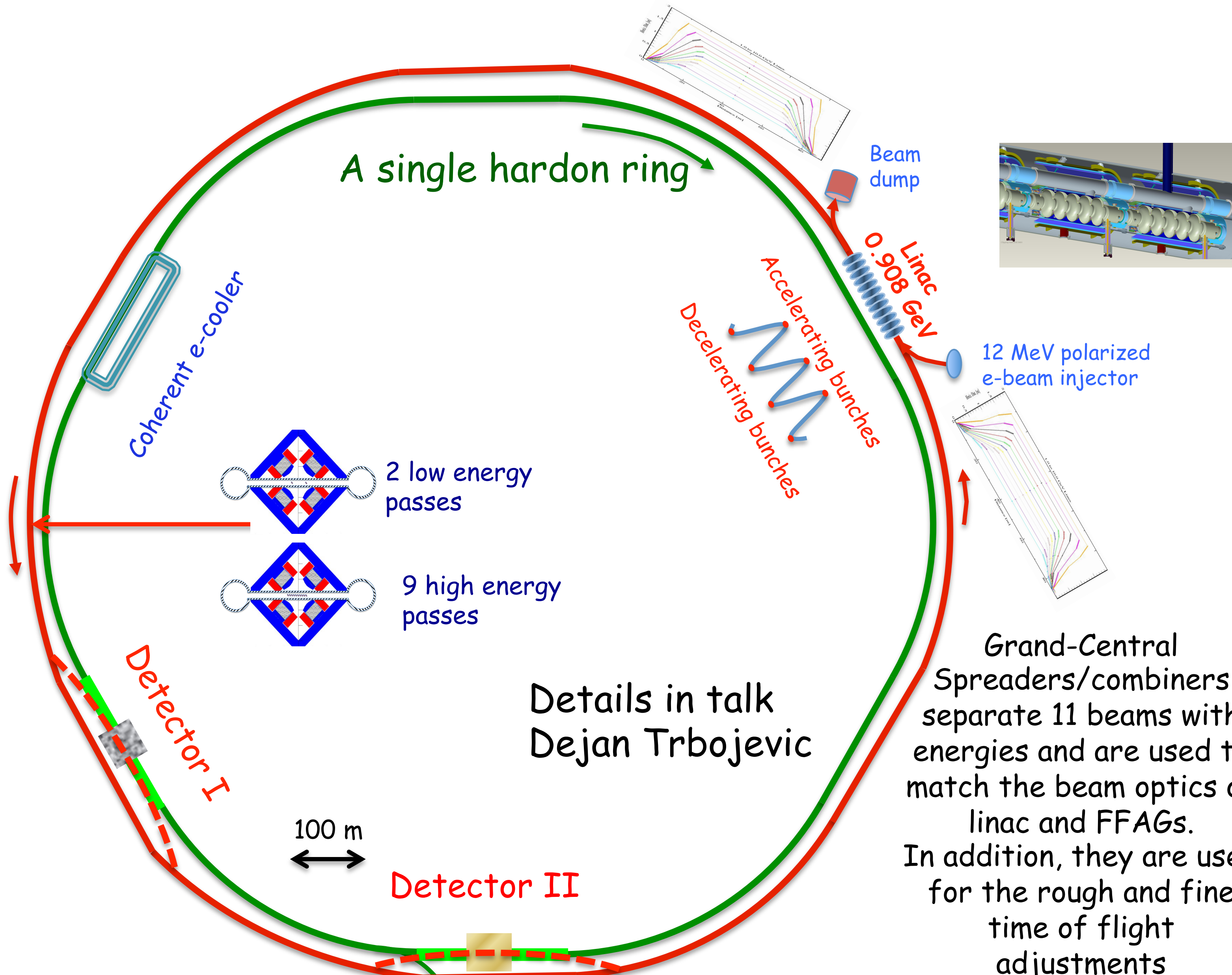
Main elements of current COST-EFFECTIVE concept

- Use ERL for electrons to reach high luminosity at high energy
- Use 11 passes through main ERL linac to reach 10 GeV
- Use 2 FFAG magnet strings in RHIC tunnel to transport 11 beams
- Consider a possibility of permanent magnet FFAG lattice
- Cool hadron beam 10-fold in all directions using coherent electron cooling (CeC) but at reduced intensity of hadron beam
- Use harmonic jump to collide electrons with 50 GeV and 250 GeV hadrons
- Use recent advances in SC quadrupole technology to design IR with β^* to 5 cm
- Use crab-crossing to avoid IR SR background
- Develop Gatling polarized electron gun with 50 mA CW current
- Build cost-effective eRHIC with luminosity exceeding 10^{33} cm^{-2} sec^{-1}



0.60 GeV
0.6 GeV
27.55 GeV
New detector
Linac 2.45 GeV
Coherent e-cooler
Polarized e-gun
0.6 GeV
Beam dump
Linac 2.45 GeV
5.50 GeV
10.4 GeV
15.3 GeV
20.2 GeV
25.1 GeV
30.0 GeV
3.05 GeV
7.95 GeV
12.85 GeV
17.75 GeV
22.65 GeV
27.55 GeV
30 GeV
ePHENIX
100 m
eSTAR
30 GeV

eRHIC with 10 GeV FFAG ERL



Grand-Central Spreaders/combiners separate 11 beams with energies and are used to match the beam optics of linac and FFAGs.
In addition, they are used for the rough and fine time of flight adjustments

eRHIC luminosity (details in Vadim's talk)

	e	p	$^{2}He^{3}$	$^{79}Au^{197}$	$^{92}U^{238}$
Energy, GeV or GeV/u	10	250	167	100	100
CM (e-u) energy, GeV		100	81.7	63.2	63.2
Number of bunches/distance between bunches	111 nsec	111	111	111	111
Bunch intensity (nucleons) ,10^{11}	0.33	0.3	0.6	0.6	0.6
Bunch charge, nC	5.3	32	10	5.2	5.2
Beam current, mA	50	420	140	67	67
Normalized rms emittance of hadrons , mm mrad		1.2	1.2	1.2	1.2
Normalized rms emittance of electrons, mm mrad	varies->	*20*	*22*	*36*	*36*
Polarization, %	70	70	70	none	none
rms bunch length, cm	0.4	5	5	5	5
β*, cm	5	5	5	5	5
Luminosity per nucleon, × 10^{33} cm^{-2}s^{-1}		1.1	2.1	1.3	1.3

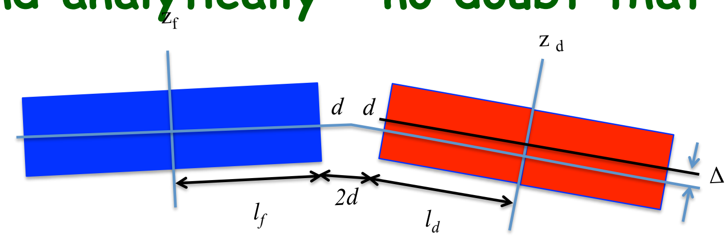
Changes in parameters: We plan using about 15% of the hadron beam intensity presently operated in RHIC. This allows to operate with present RHIC vacuum chamber (e.g. no coating is needed) and without space-charge compensation.

Changes in beam parameters compared with previous design

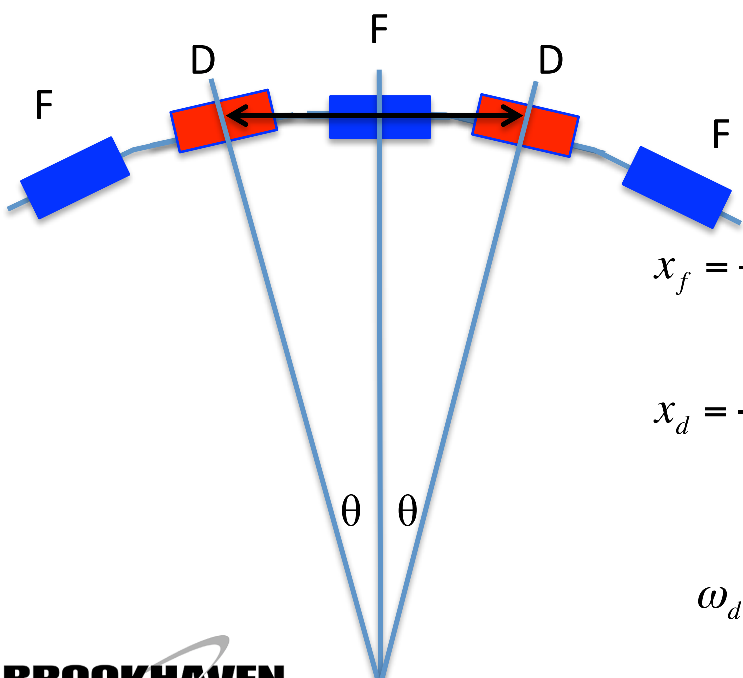
- We plan using about 15% of the hadron beam intensity presently operated in RHIC. Details are in V. Ptitsyn talk.
- This allows to operate with present RHIC vacuum chamber (e.g. no coating is needed) and without space-charge compensation.
- We reduced the collision rep-rate 1.5 fold from 14 MHz to 9 MHz to provide for a possibility to use eSTAR and ePHENIX. It eliminated a need for new (faster) RHIC injection kickers
- At the same time we increased e-beam charge per bunch 1.5-fold from 3.8 nC to 5.3 nC. We re-evaluated the wake-field for this charge per bunch.
- We reduced the frequency of SRF linac from 704 MHz to 412.8 MHz. This reduced number of cavities to ~ 30, and significantly reduced the HOM losses and the linac wake-field, improves polariztion….

Simple picture of FFAG

- An ideal eRHIC FFAG cell is comprised of two quadrupoles (F & D) whose magnetic axes are shifted horizontally with respect to each other by Δ
- The structure has a natural bilateral symmetry, e.g. all extrema (min and max) are in the centers of the quadrupoles *(independently of any approximation!)*
- Orbit dependence on the energy can be easily found in paraxial approximation
- **Everything can be done accurately and analytically – no doubt that proposed FFAG lattice would work!**



$$L=2(l_F+l_D+2d)$$



Orbit

$$x_f = -\frac{a_{xd}\theta + c_{xd}\left(\Delta + d\theta\right)}{a_{xf}c_{xd} + a_{xd}c_{xf} + 2dc_{xd}c_{xf}}$$

$$x_d = -\frac{a_{xf}\theta - c_{xf}\left(\Delta - d\theta\right)}{a_{xf}c_{xd} + a_{xd}c_{xf} + 2dc_{xd}c_{xf}}$$

$$a_{xf} = \cos\varphi_f;\ b_{xf} = \frac{\sin\varphi_f}{\omega_f};c_{xf} = -\omega_f \sin\varphi_f$$

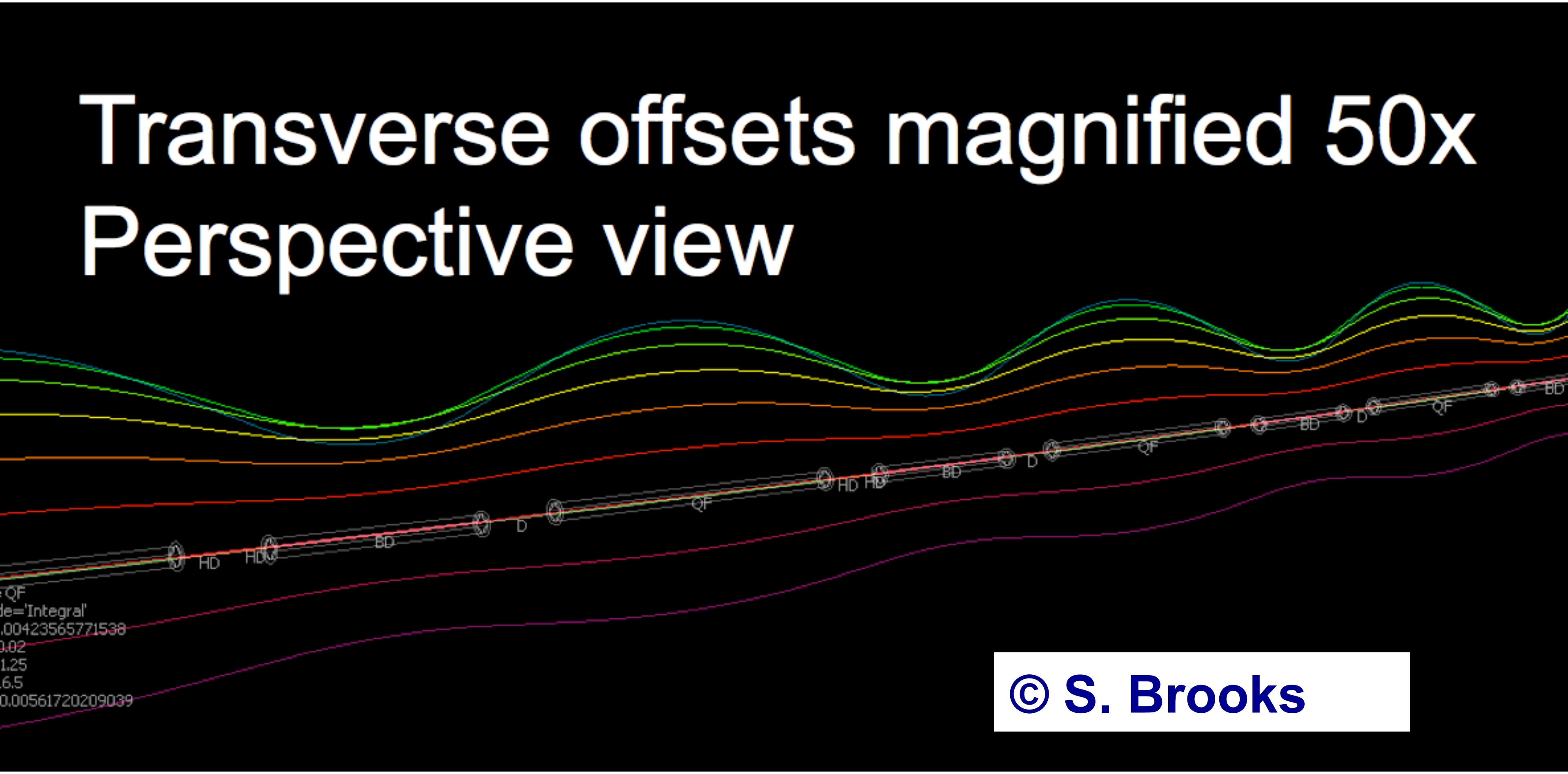
$$a_{xd} = \cosh\varphi_d;\ b_{xd} = \frac{\sinh\varphi_d}{\omega_d};c_{xd} = \omega_d \sinh\varphi_d$$

$$a_{yf} = \cosh\varphi_f;\ b_{yf} = \frac{\sinh\varphi_f}{\omega_f};c_{yf} = \omega_f \sinh\varphi_f$$

$$a_{yd} = \cos\varphi_d;\ b_{yd} = \frac{\sin\varphi_d}{\omega_d};c_{yd} = -\omega_d \sinh\varphi_d$$

$$\omega_d = \sqrt{-\frac{eG_d}{pc}};\omega_f = \sqrt{\frac{eG_f}{pc}}. \qquad \varphi_{f,d} = \omega_{f,d}\, l_{f,d}$$

Trajectories are paraxial!

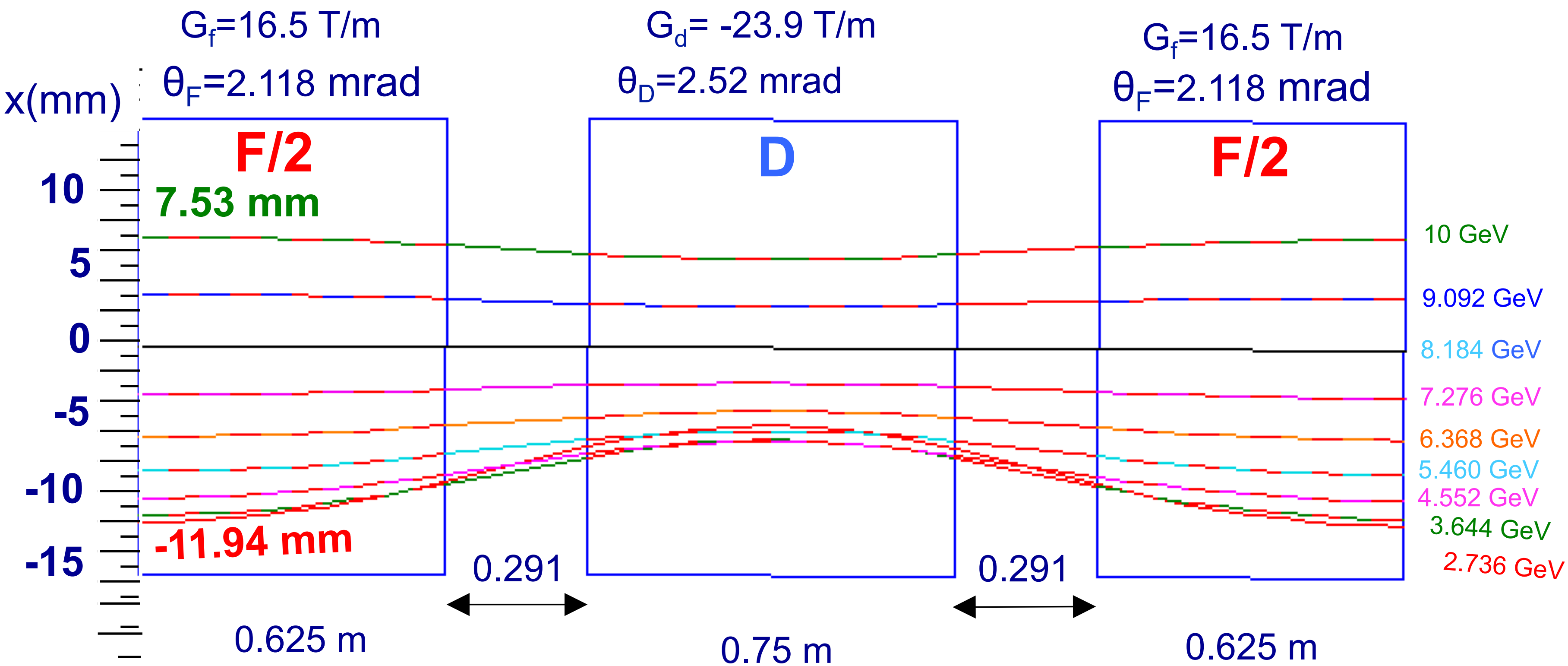


Basic Doublet Cell : 2.736 - 10 GeV, 2.582 m

© D. Trbojevic, V. Ptitsyn, S. Brooks

Trajectories are paraxial even for lowest energies!

Maximum angles are ~ 4 mrad!



This makes the analysis straightforward and definite.
The acceptance of such lattice is enormous and shooting beam into a roughly correct direction will have it carried out by FFAG.

Stability: both horizontal and vertical motion must be stable

- All calculations can be done analytically and exactly
- At the stability line $|G_d| l_d = G_f l_f$ when the focal strength of F and D quad are equal, the cell is stable at any energy above a cut-off energy

$$b(E_{\min})c(E_{\min}) = -1$$

- Off the stability line, the stability is limited at both low energy and at high energy, e.g. the operational energy range for FFAG is limited by $r = E_{\max}/E_{\max}$
- The later is a function of lattice parameters
- **There are advantages to veer of the stability line: the orbit deviations and power of synchrotron radiation can be significantly reduced**
- **Our choice for FFAG II has** $\lambda = 0.25; \varepsilon = 0.161$ **and deviated from the line by** $\delta\varepsilon = 0.034$

$$M_t = M_d D M_f = \begin{bmatrix} a_t & b_t \\ c_t & d_t \end{bmatrix}; \tilde{M}_t = M_f D M_d = \begin{bmatrix} d_t & b_t \\ c_t & a_t \end{bmatrix}$$

$$\boxed{\begin{gathered} 1 > \cos\mu_{x,y} = 1 + 2b_{x,yt}c_{x,yt} > -1; \Leftrightarrow 0 > b_{x,yt}c_{x,yt} > -1 \\ \beta_{x,yF} = \sqrt{-\frac{d_{x,yt}b_{x,yt}}{a_{x,yt}c_{x,yt}}}; \beta_{x,yD} = \sqrt{-\frac{a_{x,yt}b_{x,yt}}{d_{x,yt}c_{x,yt}}}. \end{gathered}}$$

Dimensionless Parameterization of the Cell

$$l_f = \frac{l}{2}(1+\lambda); l_d = \frac{l}{2}(1-\lambda); \;\; \varepsilon = \frac{\varphi_f - \varphi_d}{\varphi_f + \varphi_d};$$

$$\varphi = \frac{\varphi_f + \varphi_d}{2}; \varphi_f = \varphi(1+\varepsilon); \;\; \varphi_d = \varphi(1-\varepsilon);$$

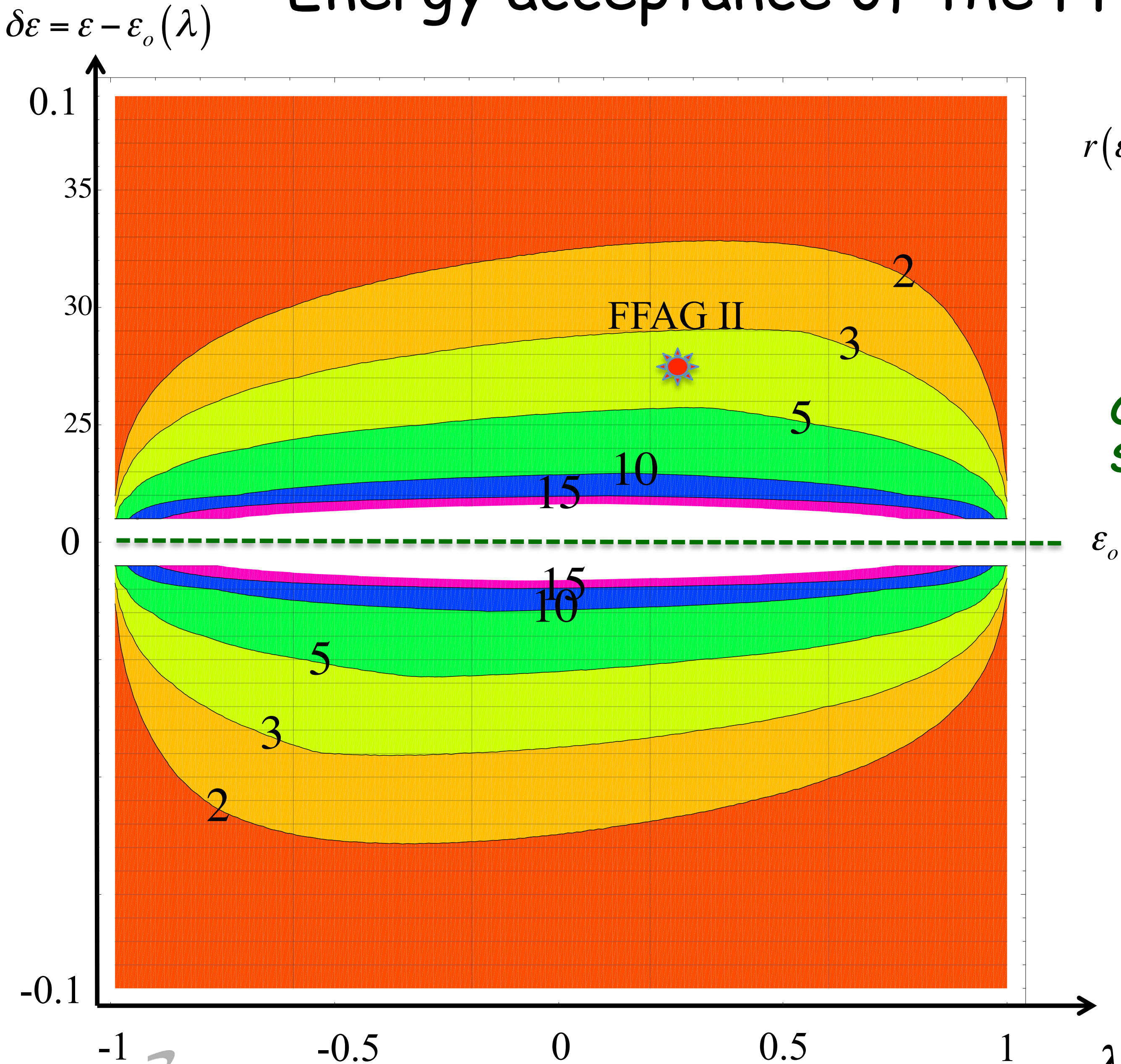
$$\omega_f = \frac{\varphi_f}{l_f} = \frac{\varphi(1+\varepsilon)}{\frac{l}{2}(1+\lambda)}; \quad \omega_d = \frac{\varphi_d}{l_d} = \frac{\varphi(1-\varepsilon)}{\frac{l}{2}(1-\lambda)};$$

Stability line

$$\lambda_o = \frac{2\varepsilon}{1+\varepsilon^2} \qquad \varepsilon = \varepsilon_o(\lambda) = \frac{1-\sqrt{1-\lambda^2}}{\lambda}$$

$$|G_d| l_d = G_f l_f$$

Energy acceptance of the FFAG



$$r(\varepsilon,\lambda) = \left(\frac{\varphi_{max}(\varepsilon,\lambda)}{\varphi_{min}(\varepsilon,\lambda)}\right)^2 = \frac{E_{max}}{E_{min}}$$

Complete Stability line

$$\varepsilon_o(\lambda) = \frac{1-\sqrt{1-\lambda^2}}{\lambda}$$

$$r(\varepsilon,\lambda) \Rightarrow \infty$$

A single FFAG 0.9 -10 GeV, r=11 ?

- We explored an attractive possibility of having a single FFAG covering all acceleration range
- Theoretically and practically it is possible
- Dejan had a lattice covering this rang
- But it has serious problems – the orbit deviation grow ~ $(E_{\max}/E_{\min})^2$
- With a fixed gradient the reducing the cell length possible but only as
- The most drastic effect is in fast increase in the power of synchrotron radiation $1/\sqrt{E_{\max}/E_{\min}}$
- After recognizing this we returned to two-ring solution, but we learned from the exercise and Vadim optimized the FFAG II lattice reducing SR power from 5 MW to about ~1.5 MW

$$r(\varepsilon,\lambda) = \left(\frac{\varphi_{\max}(\varepsilon,\lambda)}{\varphi_{\min}(\varepsilon,\lambda)}\right)^2 = \frac{E_{\max}}{E_{\min}}$$

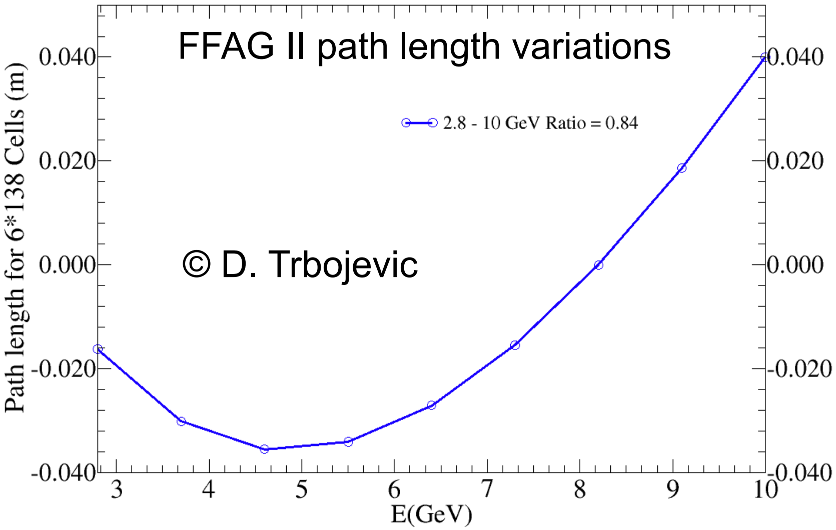
$$r >> 1$$

$$x_{orbit} \sim \frac{L^2}{R}\left(\frac{E_{\max}}{E_{\min}}\right)^2 \sim \frac{E_{\max}}{E_{\min}}$$

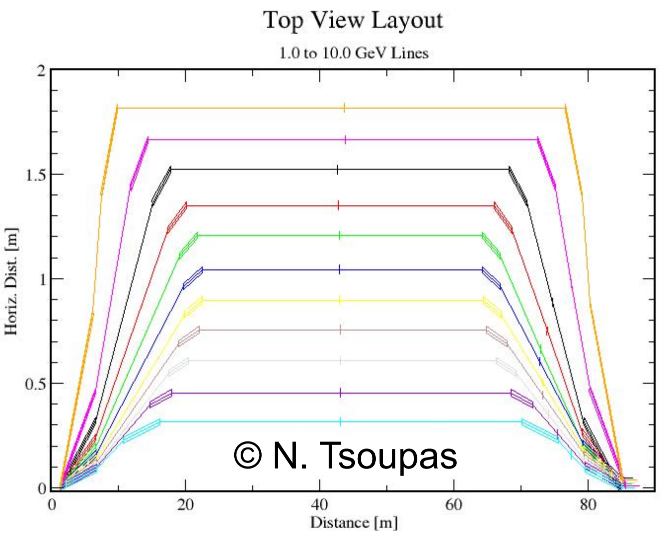
$$\frac{P_{SR}}{E^2} \sim \int B^2 ds \sim \int G^2 x_{orbit}^2 ds \sim \left(\frac{E_{\max}}{E_{\min}}\right)^2$$

Isochronicity

- ERL requires the time of flight between consecutive passes to be exactly an integer number of ERL RF periods and must be fine tuned
- Exception is the top energy, where there must be a 180-degree phase change (e.g. a half of the ERL RF period)
- FFAG is not a isochronous lattice and the pass-lengths adjustment at each energy is required
- All beams must pass the linac on axis and their envelopes (β-functons) must be matched
- We decided to use 11 individual beam-lines in the Splitter/Combiners to both to correct the time of flight, R_{56} and the optics functions
- Details are in talks by Vadim and Dejan

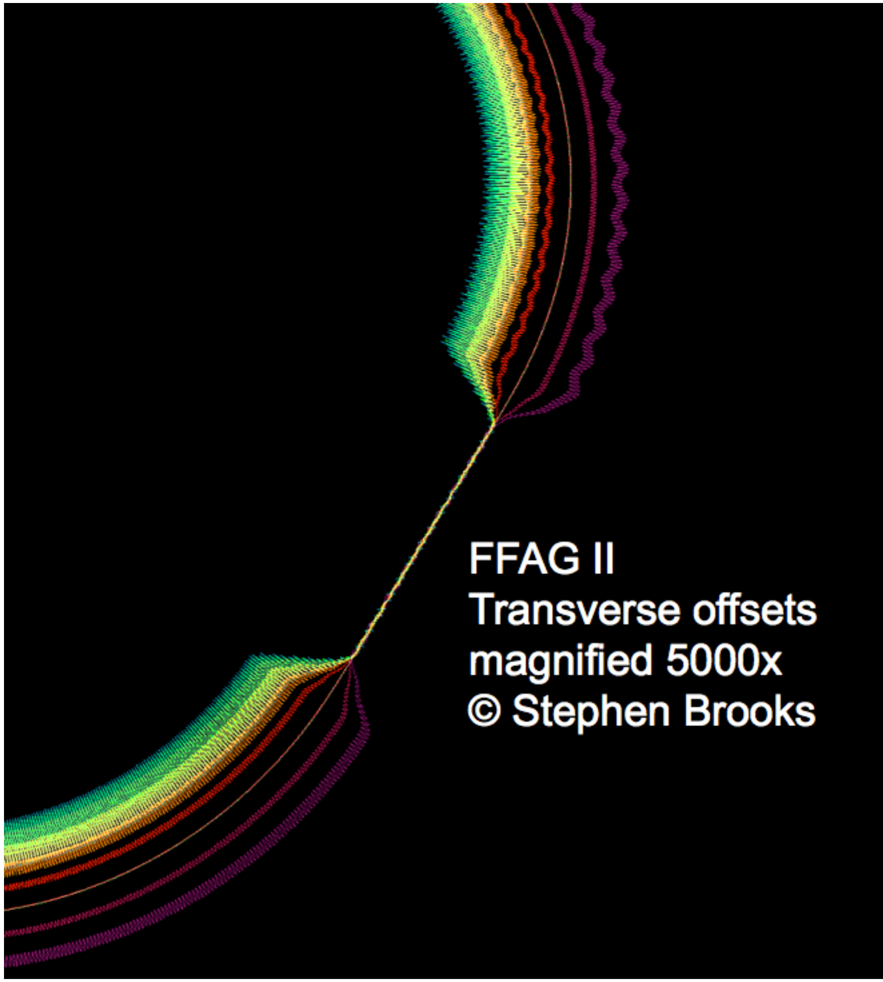


Grand-Central: splitters & combiners



Straight Sections & By-passes

- We have 6 200-m long straight section in RHIC tunnel
- One of them will used for ERL SRF linac and two are for eRHIC detectors
- RHIC arcs also have a transition area with radius of curvature different from the main arc
- Hence, we need a transitions from regular curved arcs to other structures
- Natural approach is using the paraxiality and the flexible nature of our FFAG lattice
  - Keeping the same FODO cells guarantees matching of beta-functions
  - shifting quadrupoles horizontally in a transition area we can put all beam on axis, e.g. to go through a straight section
  - or we can flip the displacements back-and-forth and use FFAG bend to go around the detectors



Details in talk by Dejan and Stephen

Beam dynamics/Systems

Studied (not necessarily finalized!):

- Optimize FFAG lattice to reduce synchrotron radiation losses
- Electron beam energy losses and energy spread caused by the interaction with the beam environment (cavities, resistive walls, pipe roughness), incoherent and coherent synchrotron radiation
- Electron beam patterns with 500 nsec gap for colliding with 100 -250 GeV hadrons

  (T FFAG I & II = T RHIC + 4 RF periods)

  - No ion accumulation in entire ERL and no FII
  - Using existing BPM technology for measuring beam position of 18 beams in FFAG II and 22 in linac and elsewhere.
- Electron beam break-up, single beam and multi-pass
- Electron beam disruption

Details in talks G.Wang and Y.Hao

Beam dynamics/ Systems

Unsolved but planned to be solved

- Complete designs of the straight sections and by-passes around detectors
- Start-to-end simulations
- Electron beam break-up, single beam and multi-pass
- De-bunching and reduction of the energy spread of the electron beam at the dump
- Effect of Linac Transients when operating with harmonic switch (e.g. at 50 GeV/n)

In progress/discussion:

Optimal energy split between FFAG I & FFAG II
Can we inject/eject 12 MeV beam into and out of main linac?
Length of the electron bunches and the need for harmonic cavities
Detailed beam dynamics with CeC
Effect of crab cavities on beam dynamics…

Killing three birds with one stone: using a gap in the bunch pattern

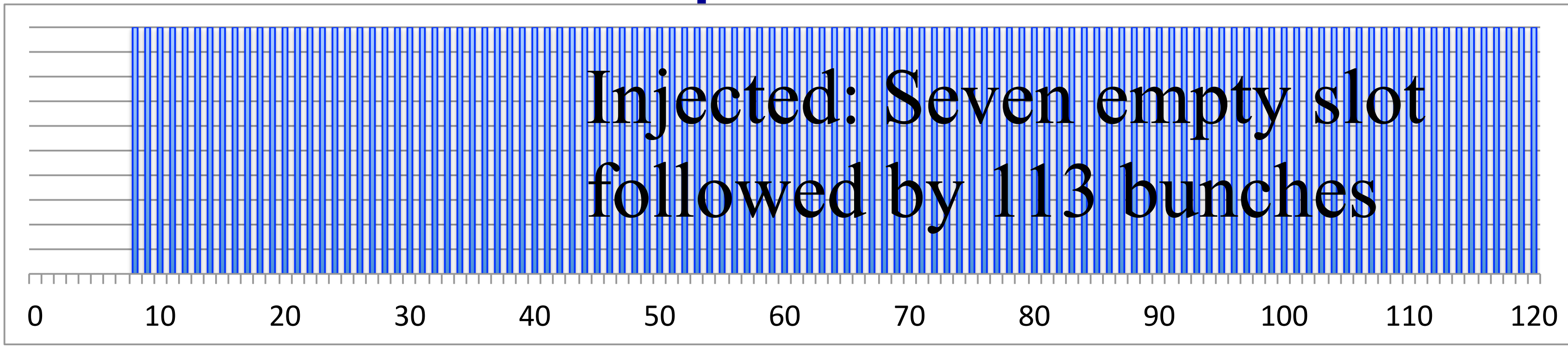
- We were looking for quite awhile to solve following problems with FFAG ERL:
  - How to avoid ion accumulation in linac where 22 beams pass through?
  - How to avoid fast ion instability?
  - How to measure (and then correct) orbits of all these beams (22 in linac, 4 in FFAG I, 18 in FFAG II, 2 in each other beam line, except detector and injection/ejection.
- Under the following conditions
  - Avoid transients (accelerating voltage variation seen by colliding bunches)
  - Avoid bunch pile-up (e.g. bunch overlap and increase of peak current!)
  - FFAGs staying within RHIC tunnel enclosure (two or single FFAG? global or local FFAG I loop)

- Detailed studies showed that using single FFAG is possible but orbit excursions and power of synchrotron radiation become prohibitive – Dropped!
- Detailed studies showed that using local low energy FFAG loop do not allow us to generate a gap without exciting serious transients. We also removed a local 100 MeV ERL-injector and decided to inject directly 12 MeV e-beam (has to be proven!)
- The only alternative solution found for measuring (if worked) would be using CCD camera and recognition system for SR image of 22 beams in FFAG II (no solution for linac!).
- Using ultra-fast (40 GHz range) digitizers would be too expensive (~$0.25M for BPM)
- Putting both FFAGs in RHIC tunnel allowed us to create regular patters with ~ 500 nsec gap: solved all three problems!

e-Beam Pattern

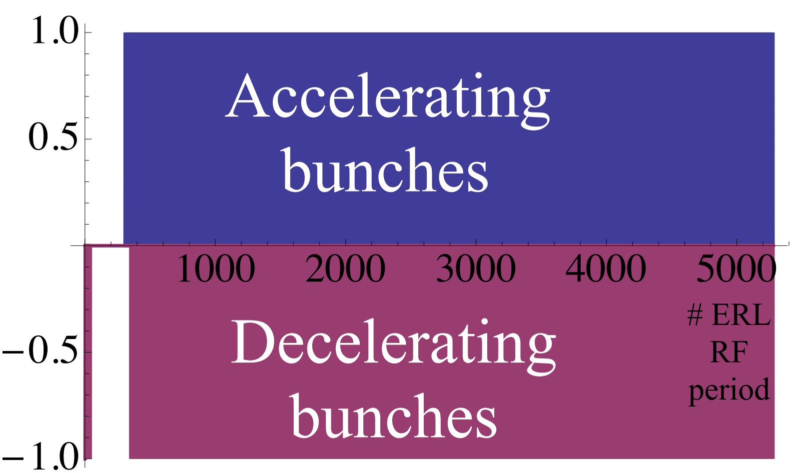
- The time structure of e-beam in ERL is driven by RHIC hadron beam: 111 of 120 filled RF buckets at ~9.38 MHz, e.g. the entire bunch pattern anywhere repeats itself with RHIC revolution frequency
- Not to lose luminosity, there must be 111 electron bunches – with equal energies and identical polarization - colliding with hadron bunches
- The most convoluted e-bunch pattern is in the linac, where 11 accelerating beams and 11 decelerating beams co-propagate: total ~ 2,500 bunches per RHIC revolution period
- We select ERL RF frequency to be a 44^{th} (all multiples of 11 work) harmonic of RHIC bunch frequency ($5{,}280^{th}$ harmonic of the RHIC revolution frequency) – the RF wavelength is ~ 70 cm)
- Both FFAGs have identical length with traveling time equal to the RHIC beam revolution time plus four ERL RF periods
- The 10 GeV pass is ½ of the ERL RF period shorter that for the rest of the energies
- In this case in the regular pattern, accelerating bunch is followed by decelerating bunch with ½ of the ERL RF period delay – hence, no transients
- The e-beam pattern has 113 bunches – two barrier bunches, #1 and #113 will experience a modestly higher accelerating voltage (total of 6.6 MeV at 10 GeV)
- The largest energy deviation (of 1.21 MeV per turn, 12.1 MeV total) will be seen by the BPM bunches

Bunch pattern: k=4

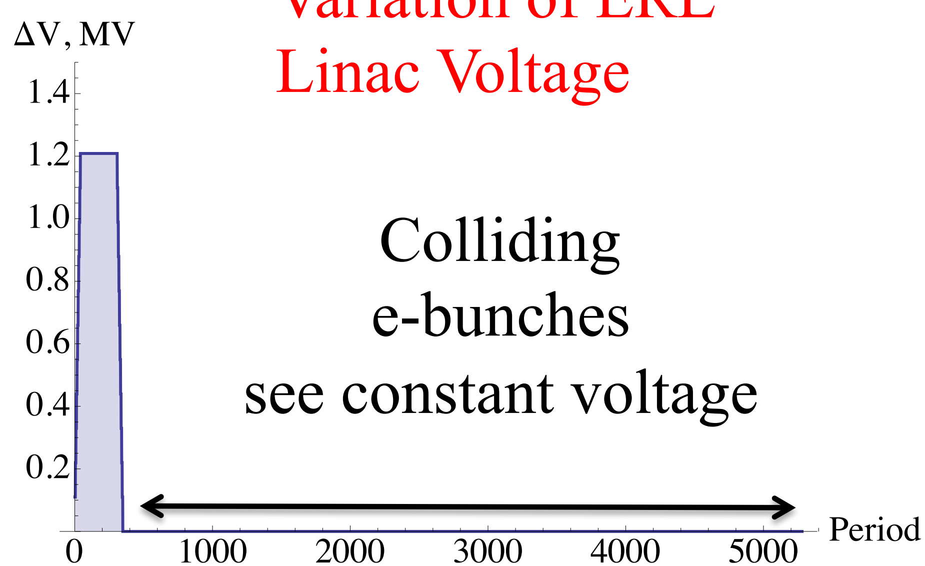


Bunch rate is equal to the rep-rate of RHIC hadron bunches (~9.38 MHz).
The bunch pattern repetition period is 44*120 = 5280 ERL RF periods (buckets)
The bunch pattern in the main linac is overlay of 22 bunch patterns shifted by 4 ERL RF periods/pass

2486 bunches in ERL Linac

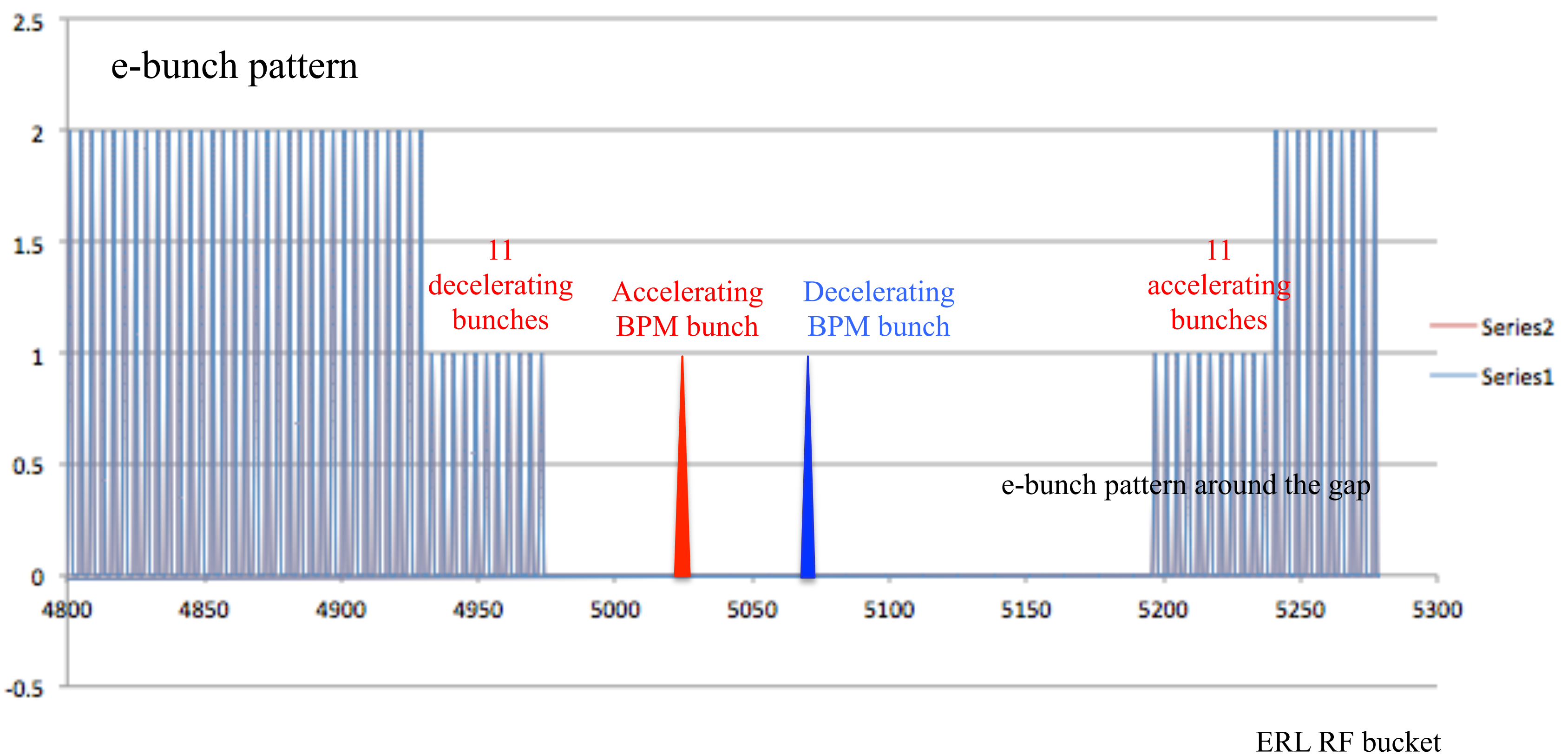


Variation of ERL Linac Voltage



e-Beam Pattern: continued

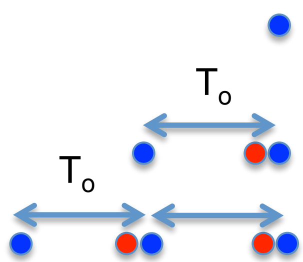
- With 500+ nsec gap the ions become unstable in every location of the ERL (see talk by Y.Hao)
- Each 11 RHIC revolution periods, e.g. with rep-rate of ~ 7 kHz, we inject an additional (BPM) bunch into the gap. Its trajectory on the trip to 10 GeV and back will be measured by single pass BPM, which we will also use for commissioning (see next slide)



Commissioning concept

Details in talk by M. Minty

- Since single accelerating bunch reduces the accelerating voltage of the RF linac by ~ 1.25 10^{-4}, it is conceivable to start commissioning with full charge bunch (5.3 nC, 4 mm RMS)
- It is sufficient for single pass BPM measurements to track the bunch orbit (as well as correct it – see details in presentation by C. Liu) as it climb up to 10 GeV and down in 22 steps.
- The minimum time between the bunch passes through the same BPM if the RHIC revolution time (e.g. ~ 12.8 μsec) leaves plenty of time to measure the orbit
- When the all passes are established, we will correct the entire orbit of the 80 km trajectory (21 x 3.8 pass through around RHIC tunnel).
- In FFAG II, we will use one BPM per 2 cells (see details for orbit correction strategy) and will start adding an additional accelerating bunch every 11 RHIC revolution periods (1320 bunch rep times in RHIC, ~ 0.14 msec). We will skip the gaps.
- We repeat this launch procedure 1242 times to establish the complete periodic structure
- After the linac transient decays out, we start injecting BPM bunches and continuously monitor and correct the orbit.

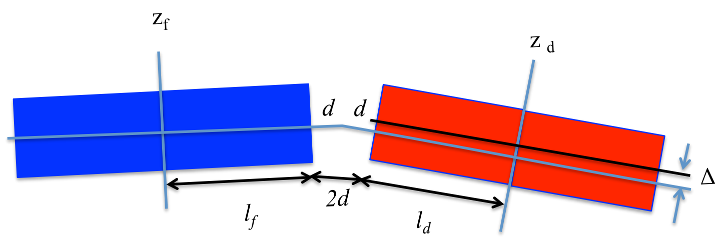


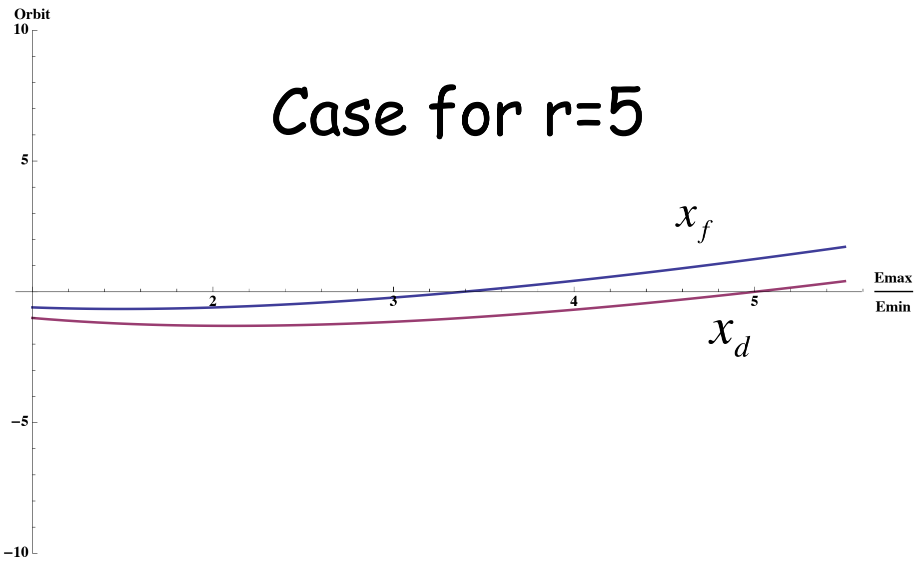
Optimizing the the SR power

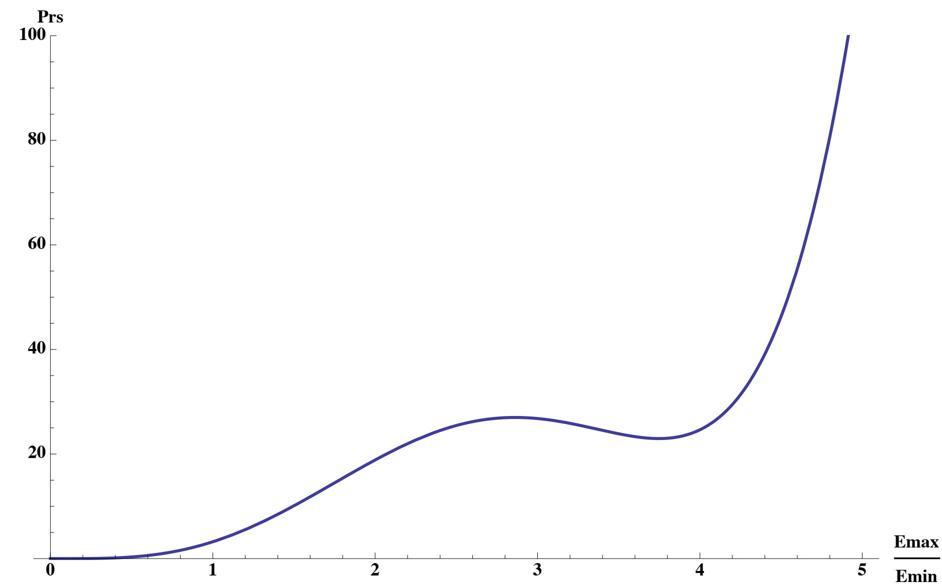
Optimization includes the cells structure &
It is very important to optimized shift between the magnetic axes of F and D quads Δ

$$x_f = -\frac{a_{xd}\theta + c_{xd}\left(\Delta + d\theta\right)}{a_{xf}c_{xd} + a_{xd}c_{xf} + 2dc_{xd}c_{xf}}$$

$$x_d = -\frac{a_{xf}\theta - c_{xf}\left(\Delta - d\theta\right)}{a_{xf}c_{xd} + a_{xd}c_{xf} + 2dc_{xd}c_{xf}}$$

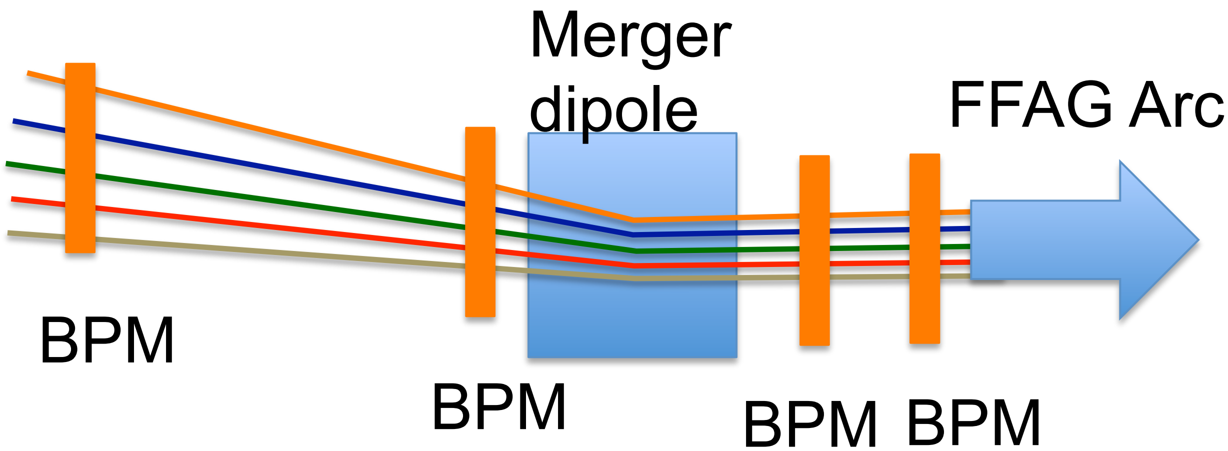






Orbit correction strategy

- Having a "BPM bunch" allows us measuring orbits individually of both accelerating and decelerating bunches (22 in linac, 17 in FFAG II and 4 in FFAG I) at all energies



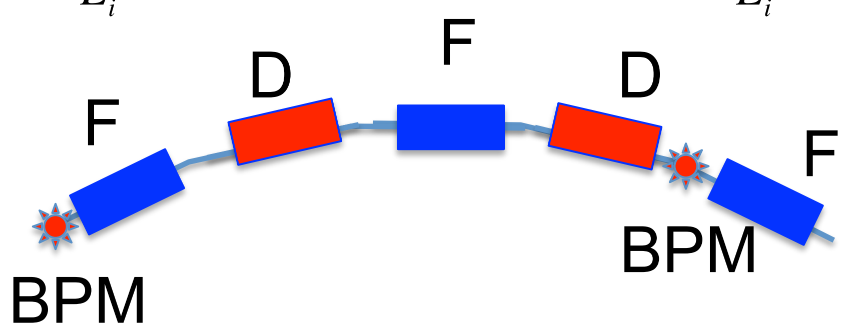
- At the exit of the Grand-Central (Splitter/Combiner) there is a merger dipole. We will install 4 BPMs (3 is minimum) to measure the energy, the positions, the angles of all bunches entering the FFAG arc
- Next step – is correcting the energies and trajectories of all beams entering the FFAG arc

Orbit correction strategy in FFAG II (17 beams with 9 distinct energies)

- All beams entering the first FFAG cell along their designed orbits
- Each FFAG cell has two shifted quadrupoles
- There are 4 unknown errors in y and 8 in x direction:
  - Y: Quads position errors X: Quads position and gradient errors

$$\Delta B_x = -G \cdot \Delta y_{quad}; \Delta B_y = G \cdot \Delta x_{quad} + x_{iquad} \cdot \Delta G;\ x_i = x_{orbit}\left(E_i\right); i = 1,2,..9$$

$$\Delta\theta^i_{y_{f,d}} = 2l_{f,d}\frac{eG_{f.d}}{E_i}\Delta y_{f,d}; \Delta\theta^i_{x_{f,d}} = 2l_{f,d}\frac{eG_{f.d}\Delta x_{f,d} + \overline{x}_{orbit}\left(E_i\right)\cdot \Delta G_{f.d}}{E_i};$$



- One BPM for two cell should work and orbit could be corrected:
  - The BPM measures 9 beam positions for 9 beam energies, which is sufficient to find both the 4 quads position and gradient errors, but also the BPM offset
  - It is obvious that correcting horizontal orbit is more challenging that vertical

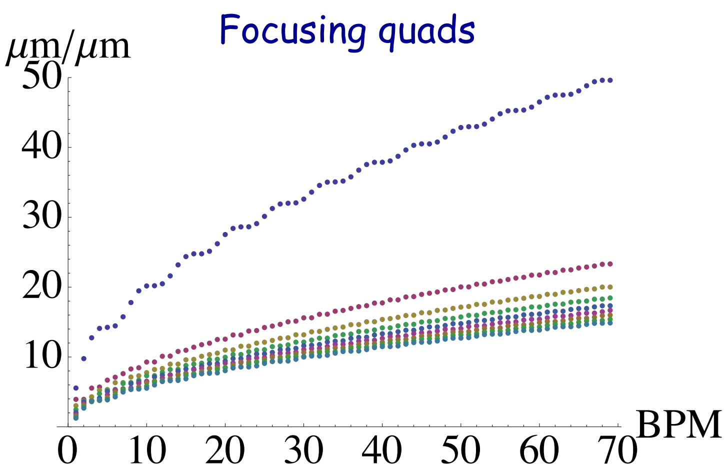
First: estimate the effect in an arc (138 FODO cells)

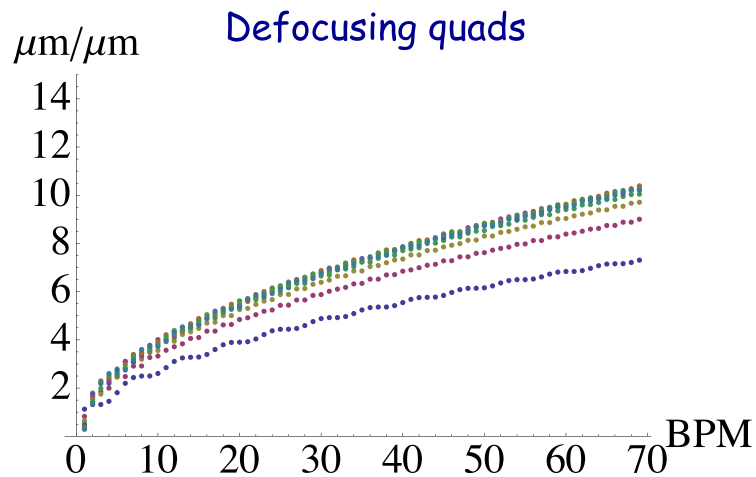
- The transport matrix (70x 9, 2x138) can be easily calculated analytically from a center of every quad to every BPM downstream at all 9 energies
- Doing it with Mathematica is straightforward

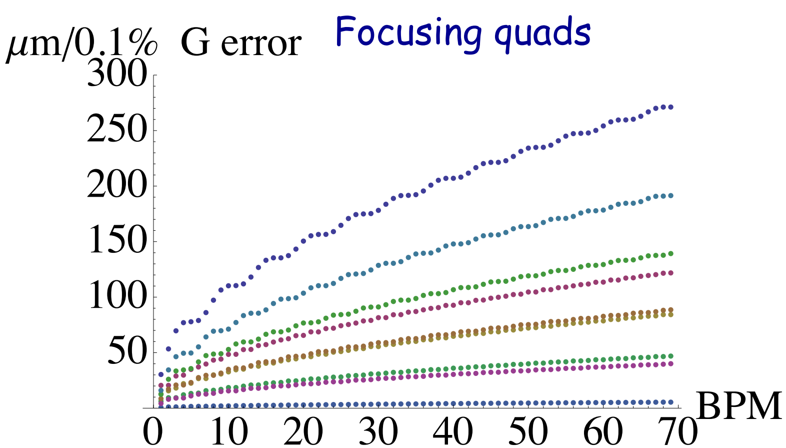
```
ω[G_, bro_] := N[Sqrt[Abs[G] / bro], 50];
f[w_, l_] := N[{{Cos[w * l], Sin[w * l] / w}, {-Sin[w * l] * w, Cos[w * l]}}, 50]
d[w_, l_] := N[{{Cosh[w * l], Sinh[w * l] / w}, {Sinh[w * l] * w, Cosh[w * l]}}, 50]
o[l_] := N[{{1., l}, {0, 1}}, 50]

 m1[bro_] := o[10].d[ω[Gd, bro], ld]
 m2[bro_] := o[10].d[ω[Gd, bro], 2 * ld].o[2 * 10].f[ω[Gf, bro], lf]
 mc[bro_] := o[10].d[ω[Gd, bro], 2 * ld].o[2 * 10].f[ω[Gf, bro], 2 * lf].o[10]
 m3[bro_] := mc[bro].o[10].m1[bro]
 m4[bro_] := mc[bro].o[10].m2[bro]
 mn1[bro_, n_] := MatrixPower[mc[bro], n].m1[bro]
 mn2[bro_, n_] := MatrixPower[mc[bro], n].m2[bro]

ncells = 138; nst = ncells / 2;
m12d = Table[0, {i, 1, 9}, {j, 1, nst}]; m12f = Table[0, {i, 1, 9}, {j, 1, nst}];
Do[m12d[[i, j]] = mn1[ Bρ[[i]], j - 1][[1, 2]] * 2 * ld / Bρ[[i]], {i, 1, 9}, {j, 1, nst}]
Do[m12f[[i, j]] = mn2[ Bρ[[i]], j - 1][[1, 2]] * 2 * lf / Bρ[[i]], {i, 1, 9}, {j, 1, nst}]
MatrixForm[m12d];
MatrixForm[m12f];
```

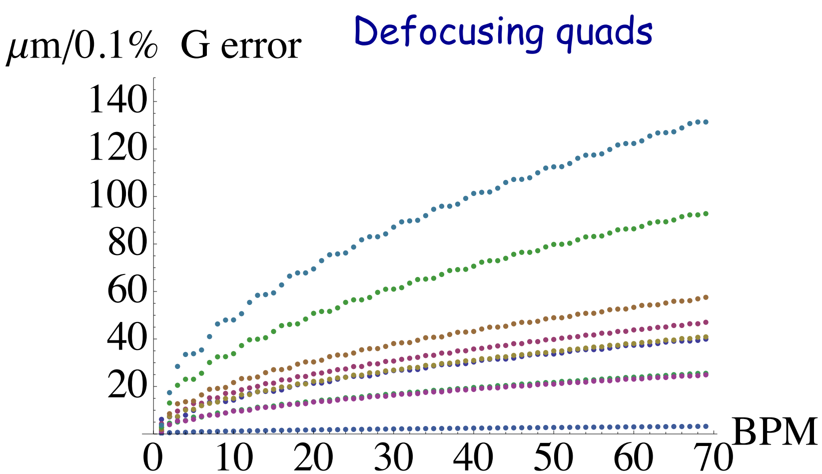
Positions and gradient errors

Horizontal orbit without corrections









Without correction, 100 μm RMS quad position errors lead to < 5 mm RMS orbit error at the arc's end. The RMS errors of 0.1 % in quads gradients max 0.31 mm RMS orbit displacement. Combination of RMS error of 0.5% in gradient and 100 μm in position will result in less that ~5mm RMS orbit errors

2 FFAG Cell Orbit correction

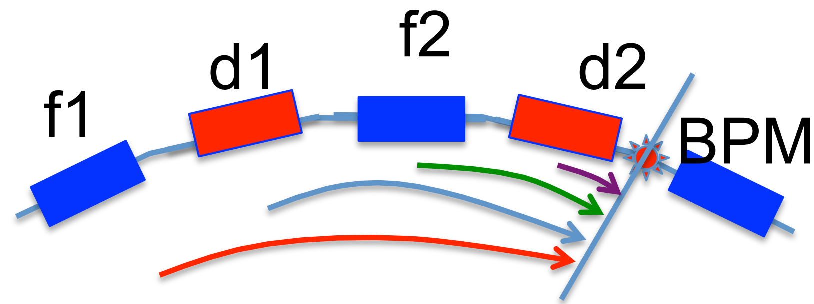
- I will talk only about horizontal orbit correction (vertical is easy!)
- BPM is measuring the error of x-position for each beam energy relative to its design orbit (which is function of energy)

$$X_o = X_m - X_{orbit}$$

- Correcting orbit locally

$$X = X_o - R\cdot Y;$$

$$X_9 = \begin{bmatrix} x(E_1) - x_{orbBPM}(E_1) \\ x(E_2) - x_{orbBPM}(E_2) \\ .. \\ x(E_9) - x_{orbBPM}(E_9) \end{bmatrix}; Y_9 = \begin{bmatrix} \Delta x_{f1} \\ \Delta G_{f1} \\ .... \\ \Delta G_{d2} \\ \Delta G_{d2} \end{bmatrix};$$



Calculate the response matrix
From 4 quads for 9 energies

$$R_{9x8} = \begin{bmatrix} M_{12}(E_1, s_{f1}, s_{BPM}) & x_{f\ orb}(E_1)\cdot M_{12}(E_1, s_{f1}, s_{BPM}) & & \\ & .... & ...... & \\ & & M_{12}(E_9, s_{d2}, s_{BPM}) & x_{d\ orb}(E_9)\cdot M_{12}(E_9, s_{d2}, s_{BPM}) \end{bmatrix}$$

SRV pseudo-inverse Correction Algorithm

$$X = X_o + R \cdot Y \Rightarrow F = \frac{1}{2} X^T X \equiv \frac{1}{2}\left(X_o^T X_o + 2Y^T \cdot R^T X_o + Y^T \cdot R^T R \cdot Y \right)$$

$$\frac{\partial F}{\partial Y^T} = R^T R \cdot Y + R^T X_o = 0 \Rightarrow \left[R^T R \right] \cdot Y = -R^T X_o$$

- $\left[R^T R \right]$ is positively defined matrix with all non-negative Eigen values
- It allows orthogonal decomposition $\left[R^T R \right]_{n \times n} = A^T \cdot \Lambda \cdot A; \quad A^T \cdot A = \hat{1}; \Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & \lambda_n \end{bmatrix}$
- If its determinant is non-zero is can be either inversed or pseudo-inversed using k eigen vectors/values

$$\left[R^T R \right]^{-1} = A^T \cdot \Lambda^{-1} \cdot A; \quad \left[R^T R \right]_k^{-1} = A^T \begin{bmatrix} \lambda_1^{-1} & 0 & 0 & 0 & 0 \\ 0 & ... & 0 & 0 & 0 \\ 0 & 0 & \lambda_k^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} A; \quad \Rightarrow Y = -\left[R^T R \right]_k^{-1} R^T X_o$$

- Using reduced number of eigen vectors frequently allows reducing sensitivity to errors and providing for a robust well-converging correction

What to expect from correction?

- Having 9 beams with different energies passing the structure differs from a regular single beam's random BPM positions
- The beam positions at a single BPM are strongly correlated and are result of the magnets errors Y_o and BPM measurement errors δX_o ,e.g.

$$X_o = R \cdot Y_o + \delta X_o$$

- Then the question is: to what accuracy we can reconstruct these errors

$$Y = -\left[R^T R\right]_k^{-1}\left(R^T R \cdot Y_o + R^T \delta X_o\right) \Rightarrow \delta Y = Y + Y_o = \left(\hat{1} - \left[R^T R\right]_k^{-1}\left[R^T R\right]\right) + \left[R^T R\right]_k^{-1} \cdot R^T \delta X_o$$

- The norm of $\boxed{E_Q = \hat{1} - \left[R^T R\right]_k^{-1}\left[R^T R\right]}$ gives us accuracy of conversion in determining the quadrupole errors
- The norm of $\boxed{E_{BPM} = \left[R^T R\right]_k^{-1} R^T}$ gives us sensitivity to BPM measurement errors
- While formally in a case of non-zero determinant (as in our case) regular inversing the matrix (k=8 in our case) gives perfect accuracy of conversion (albeit subject to errors in response matrix) $E_Q = \hat{1} - \left[R^T R\right]^{-1}\left[R^T R\right] \equiv 0$, in practice the norm of $E_{BPM} = \left[R^T R\right]^{-1} R^T$ can go through the roof…. In short, choice of k for pseudo-inverse depends on a specific system.

Numbers

$$R_{9x8} = \begin{pmatrix} 0.0470643 & -0.000457596 & 0.336476 & -0.0018391 & -0.0676433 & 0.00065768 & -0.655307 & 0.00358175 \\ 0.0345343 & -0.000365636 & 0.238229 & -0.0012445 & 0.0300737 & -0.000318408 & -0.0722839 & 0.000377608 \\ 0.0272633 & -0.000288036 & 0.183212 & -0.0007715 & 0.0587495 & -0.000620689 & 0.098643 & -0.000415383 \\ 0.0225179 & -0.000220413 & 0.148387 & -0.000377306 & 0.0666658 & -0.000652547 & 0.152895 & -0.000388771 \\ 0.0191781 & -0.000160431 & 0.124483 & -0.0000389446 & 0.0672919 & -0.000562919 & 0.167446 & -0.0000523854 \\ 0.0167003 & -0.000106616 & 0.107111 & 0.00025729 & 0.0651993 & -0.000416237 & 0.167103 & 0.000401397 \\ 0.0147891 & -0.0000579329 & 0.0939383 & 0.000520315 & 0.0621394 & -0.000243417 & 0.161043 & 0.000892001 \\ 0.0132702 & -0.0000136088 & 0.0836183 & 0.000756328 & 0.0588321 & -0.0000603333 & 0.15297 & 0.00138361 \\ 0.0120341 & 0.0000269615 & 0.0753215 & 0.000969859 & 0.0555812 & 0.000124526 & 0.144456 & 0.00186005 \end{pmatrix}$$

8 Eigen values span 18 orders of magnitude

$$\left[R^T R\right]$$

Λ: { 0.648201, 0.257988, 0.000187647, 1.85522E-6, 2.50949E-10, 6.41705E-14, 2.23559E-16, 5.72114E-19 }

It means that orbits are strongly correlated!
One can just guess that k=3 will do the job – and it does*!

$$\boxed{E_{BPM} = \left[R^T R\right]_k^{-1} R^T}$$

$$\left[R^T R\right]_3^{-1} R^T = \begin{pmatrix} 0.443925 & -0.490352 & -0.318178 & -0.055646 & 0.152999 & 0.301741 & 0.404665 & 0.475159 & 0.5231 \\ 0.257106 & -0.483212 & -0.338251 & -0.121289 & 0.0518971 & 0.17641 & 0.263563 & 0.324155 & 0.366177 \\ -3.7868 & 9.41155 & 6.74516 & 2.79432 & -0.36411 & -2.64265 & -4.24503 & -5.36581 & -6.14915 \\ 2.1606 & -4.04329 & -2.82865 & -1.01076 & 0.440227 & 1.48333 & 2.21338 & 2.72087 & 3.07276 \\ 19.8742 & -36.9655 & -25.7654 & -9.0587 & 4.26516 & 13.8377 & 20.5333 & 25.1844 & 28.4067 \\ 0.838815 & -1.56674 & -1.09679 & -0.392702 & 0.1694 & 0.573539 & 0.856418 & 1.05308 & 1.18946 \\ -5.43717 & 8.4237 & 6.16018 & 2.45128 & -0.556536 & -2.73761 & -4.27475 & -5.35062 & -6.10233 \\ 4.22606 & -7.87592 & -5.51007 & -1.96687 & 0.86147 & 2.89478 & 4.31785 & 5.30707 & 5.99297 \end{pmatrix}$$

$$Max\left[R^T R\right]_3^{-1} R^T = 28.4$$

*k=8 gives humongous sensitivity to BPM errors and is completely useless!!

k=3

Conversion of Quad errors is excellent

$$R-R\left[R^T R\right]_3^{-1}\left[R^T R\right]=\begin{pmatrix} 1.26703\times10^{-7} & -9.14954\times10^{-6} & -8.0737\times10^{-6} & -0.0000605584 & 0.0000375341 & -0.0000510178 & -8.77217\times10^{-6} & -0.000153405 \\ -4.34122\times10^{-7} & 0.0000344706 & 0.0000305917 & 0.000229625 & -0.000142277 & 0.000190945 & 0.0000332485 & 0.000581935 \\ -5.69492\times10^{-8} & -2.01419\times10^{-6} & -2.26846\times10^{-6} & -0.0000174058 & 0.0000106832 & -7.73854\times10^{-6} & -2.48803\times10^{-6} & -0.0000449109 \\ 2.01229\times10^{-7} & -0.0000241573 & -0.000021469 & -0.000161372 & 0.0000999248 & -0.000133428 & -0.0000233497 & -0.000408768 \\ 4.36275\times10^{-7} & -0.0000279674 & -0.0000243513 & -0.000182416 & 0.000113121 & -0.000158255 & -0.0000264435 & -0.000461496 \\ 4.80261\times10^{-7} & -0.0000191748 & -0.0000163698 & -0.000122103 & 0.0000758655 & -0.000111046 & -0.0000177418 & -0.00030869 \\ 2.57602\times10^{-7} & -2.72115\times10^{-6} & -1.98692\times10^{-6} & -0.0000142538 & 9.01866\times10^{-6} & -0.0000183971 & -2.11685\times10^{-6} & -0.0000358392 \\ -2.25373\times10^{-7} & 0.0000180943 & 0.0000159033 & 0.000119324 & -0.0000739403 & 0.000101268 & 0.0000172814 & 0.000302026 \\ -9.26898\times10^{-7} & 0.0000411922 & 0.000035521 & 0.000265313 & -0.000164756 & 0.000235973 & 0.0000385226 & 0.00067131 \end{pmatrix}$$

$$Max\left(R-R\left[R^T R\right]_3^{-1}\left[R^T R\right]\right)=6.7\cdot10^{-4}$$

Thus: orbit correction based on at one BPM per two cells total of 70 per arc):

orbit measurements with RMS BMP errors of 10 µm for all nine beams (energies)
k=3 (SVD) will lead to

Residual RMS orbit errors at BPM ~ 20 µm;
F quad position determined with accuracy of 10 µm
D-quad position determined with accuracy of 30 µm
Gradient errors determined the with accuracy of ~ 10 ppm (1E-5).

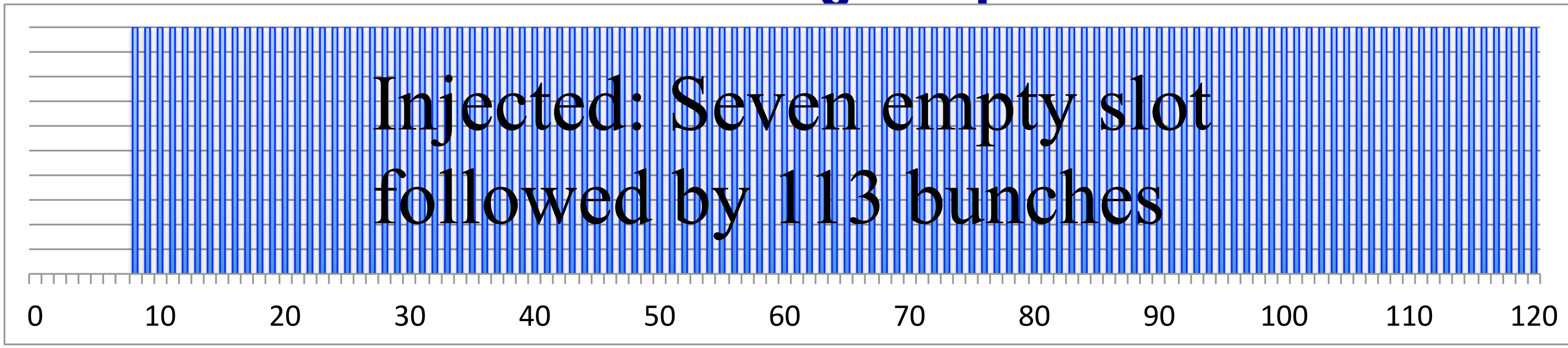
Such correction can be rolled along the beam-line or/ and extended to using 2 BPMs and 4 cells, etc… more in talk by Chuyu

Conclusions

- We are considering 11-pass ERL with FFAG arcs as a cost effective solution for 10 GeV ERL for eRHIC
- The study is incomplete!
- Today we are giving you a snap-shot of the study
- Many question remain unanswered and many systems have to be designed and tuned, but…
- We made a lot of progress and address a number of major challenges and did not find any showstoppers
- Most importantly we developed a clear strategy of measuring and controlling orbits of all 22 beams in the ERL and evaluating the FFAG lattice
- We also found, as was long expected by Dejan Trbojevic, that FFAG is a very simple and robust transport line

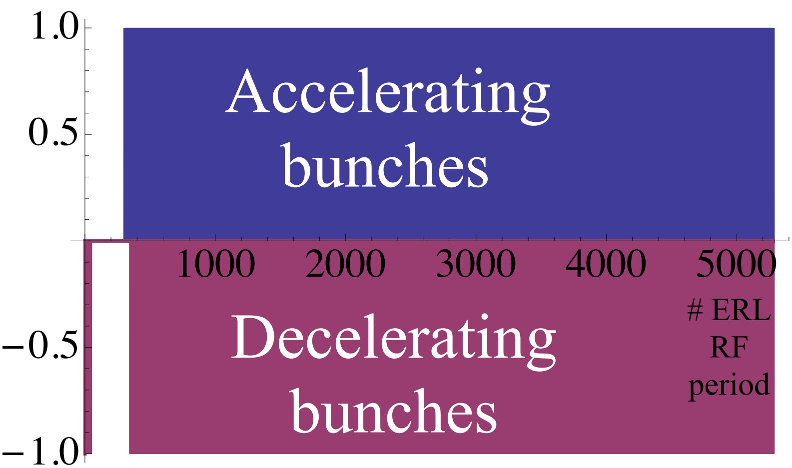
……BACK-UPS………

Harmonic jump: k=3

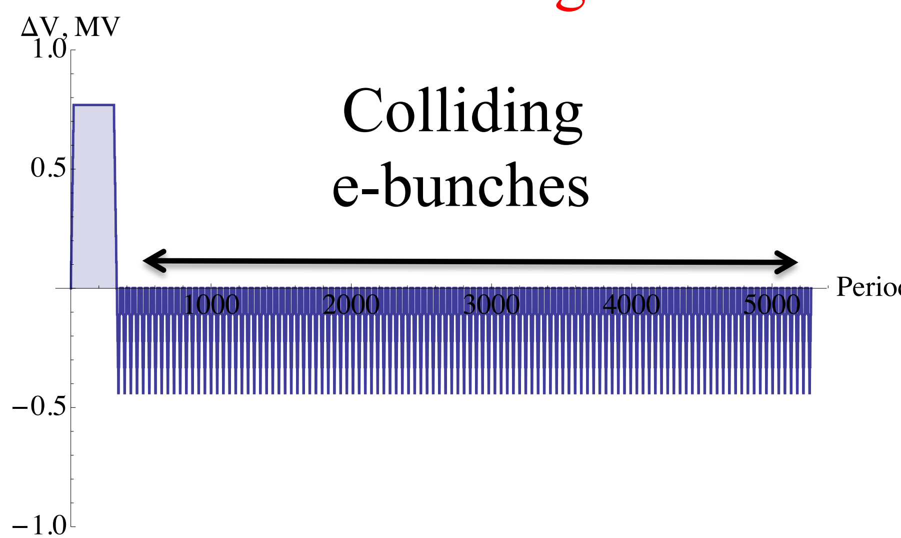


Unsolved problem

2486 bunches in ERL Linac



Variation of ERL Linac Voltage

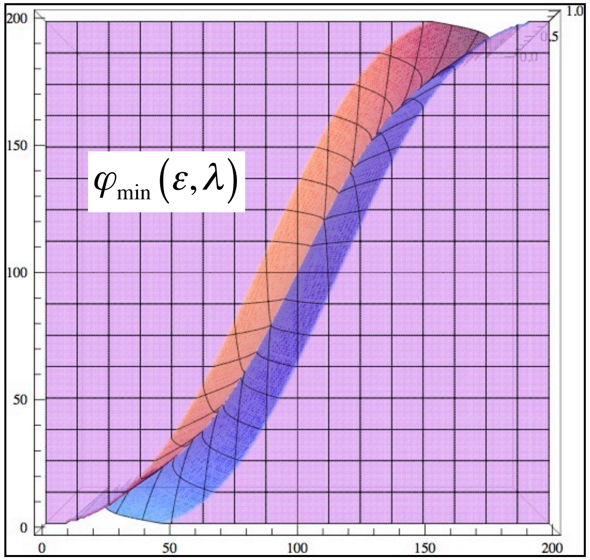


Minimum

$$M_t = M_d D M_f = \begin{bmatrix} a_t & b_t \\ c_t & d_t \end{bmatrix}; \tilde{M}_t = M_f D M_d = \begin{bmatrix} d_t & b_t \\ c_t & a_t \end{bmatrix}$$

$$1 > \cos\mu_{x,y} = 1 + 2b_{x,yt}c_{x,yt} > -1; \Leftrightarrow 0 > b_{x,yt}c_{x,yt} > -1$$

$$\beta_{x,yF} = \sqrt{-\frac{d_{x,yt}b_{x,yt}}{a_{x,yt}c_{x,yt}}}; \beta_{x,yD} = \sqrt{-\frac{a_{x,yt}b_{x,yt}}{d_{x,yt}c_{x,yt}}}.$$



Dimensionless Parameterization of the Cell

$$l_f = \frac{l}{2}(1+\lambda); l_d = \frac{l}{2}(1-\lambda);$$

$$\varphi = \frac{\varphi_f + \varphi_d}{2}; \varepsilon = \frac{\varphi_f - \varphi_d}{\varphi_f + \varphi_d};$$

$$\varphi_f = \varphi(1+\varepsilon); \quad \varphi_d = \varphi(1-\varepsilon);$$

$$\omega_f = \frac{\varphi_f}{l_f} = \frac{\varphi(1+\varepsilon)}{\frac{l}{2}(1+\lambda)}; \quad \omega_d = \frac{\varphi_d}{l_d} = \frac{\varphi(1-\varepsilon)}{\frac{l}{2}(1-\lambda)};$$

$$G_f = G\left(\frac{1+\varepsilon}{1+\lambda}\right)^2; G_d = -G\left(\frac{1-\varepsilon}{1-\lambda}\right)^2;$$

Stability line

$$|G_d| l_d = G_f l_f \quad \lambda_o = \frac{2\varepsilon}{1+\varepsilon^2} \quad \varepsilon = \varepsilon_o(\lambda) = \frac{1-\sqrt{1-\lambda^2}}{\lambda}$$

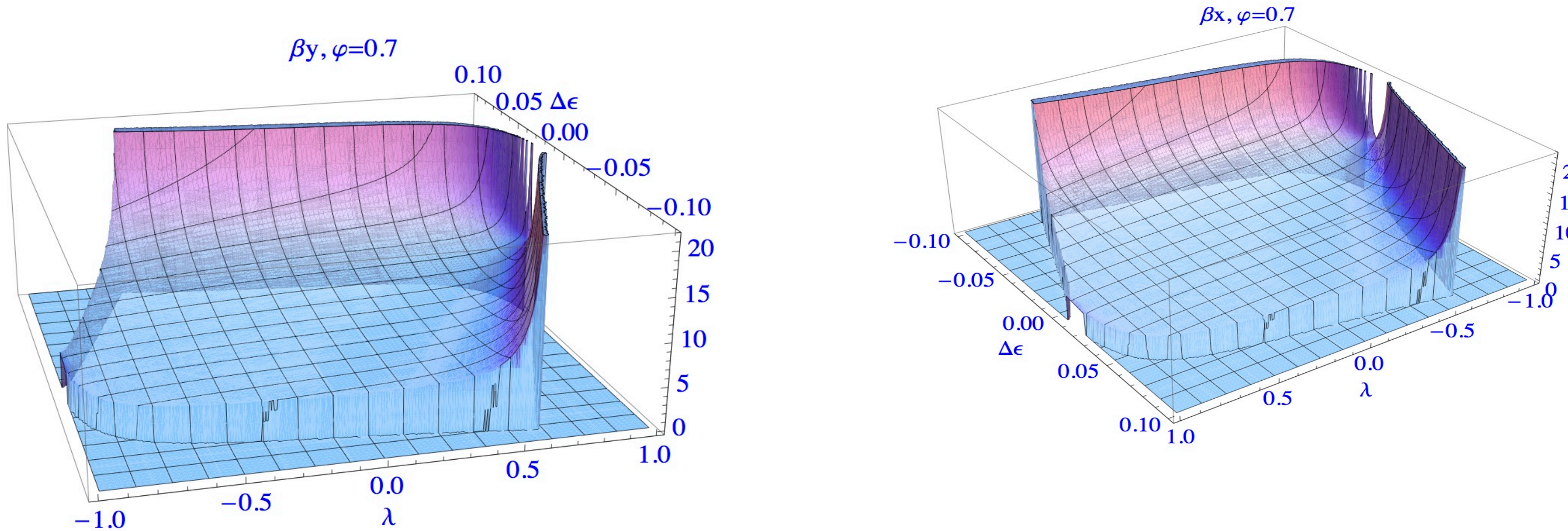
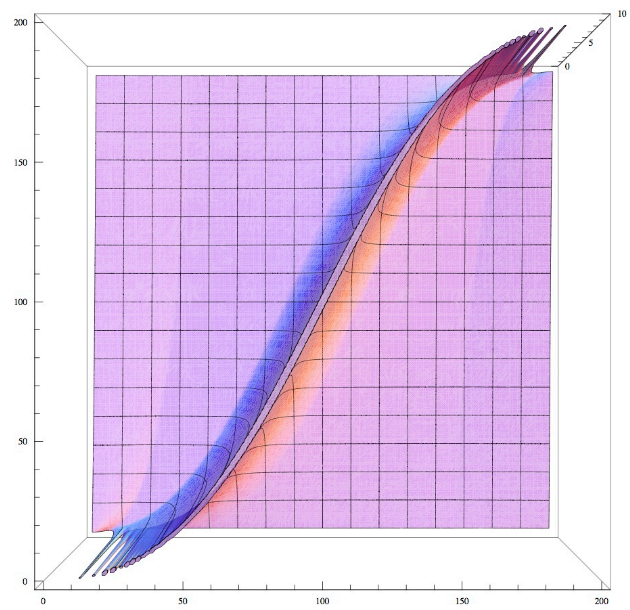


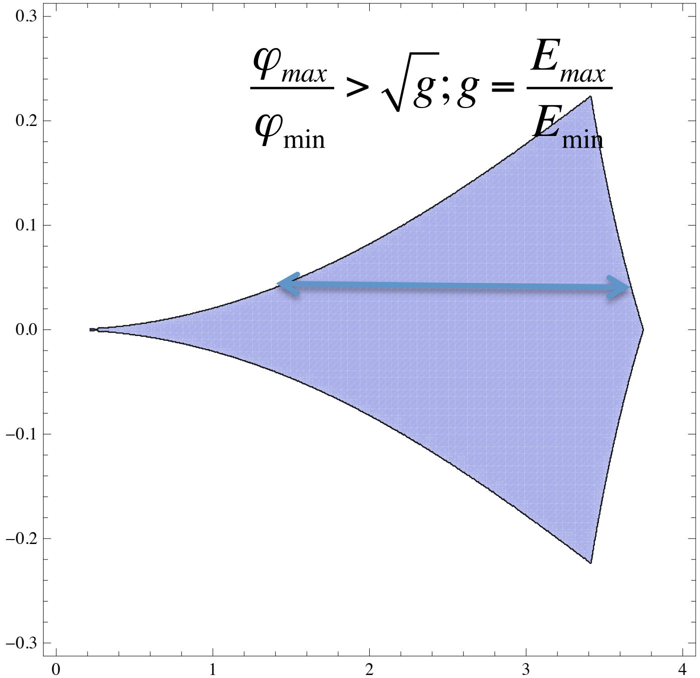
Fig. 14. Values of β_{xf}, β_{yd} in the centers of quads (i.e. maxima for the cell) around the stability line for $\varphi = 0.7$. One can see that it turns to zero where beam is unstable.



Top view 3D-plot for $r(\varepsilon,\lambda)$: $\lambda = 0.01\cdot x - 1$ is the horizontal axis and $\varepsilon = 0.01\cdot y - 1$ is the vertical axis. The drops at the end points are superficial because I used limited grid and some points fall of the "fault line". Clipping is done at $r=10$.

All distances in units of Lθ

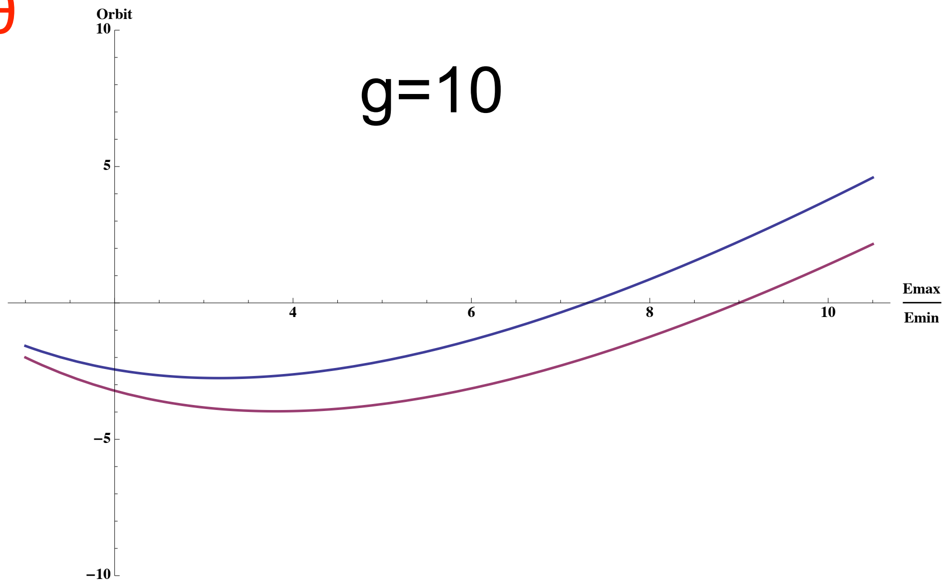
Length asymmetry



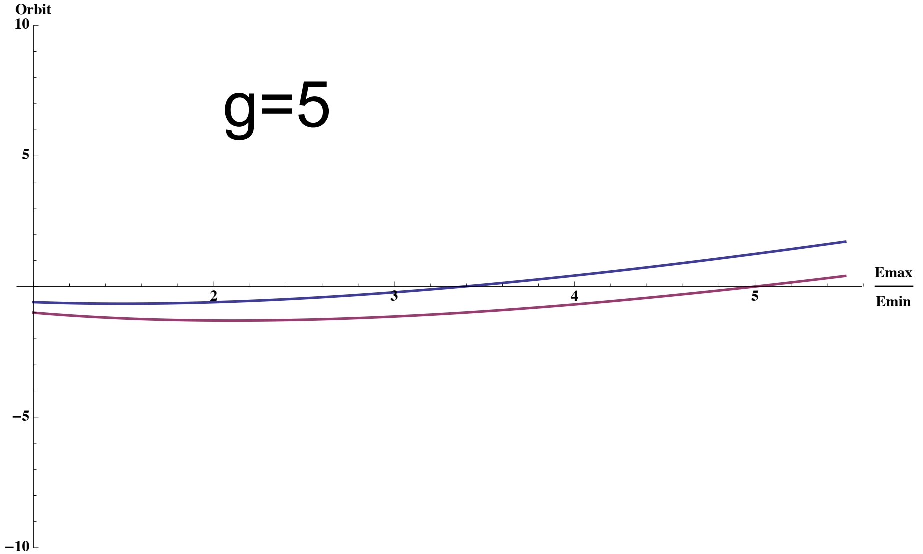
$$\frac{\varphi_{max}}{\varphi_{\min}} > \sqrt{g}; g = \frac{E_{max}}{E_{\min}}$$

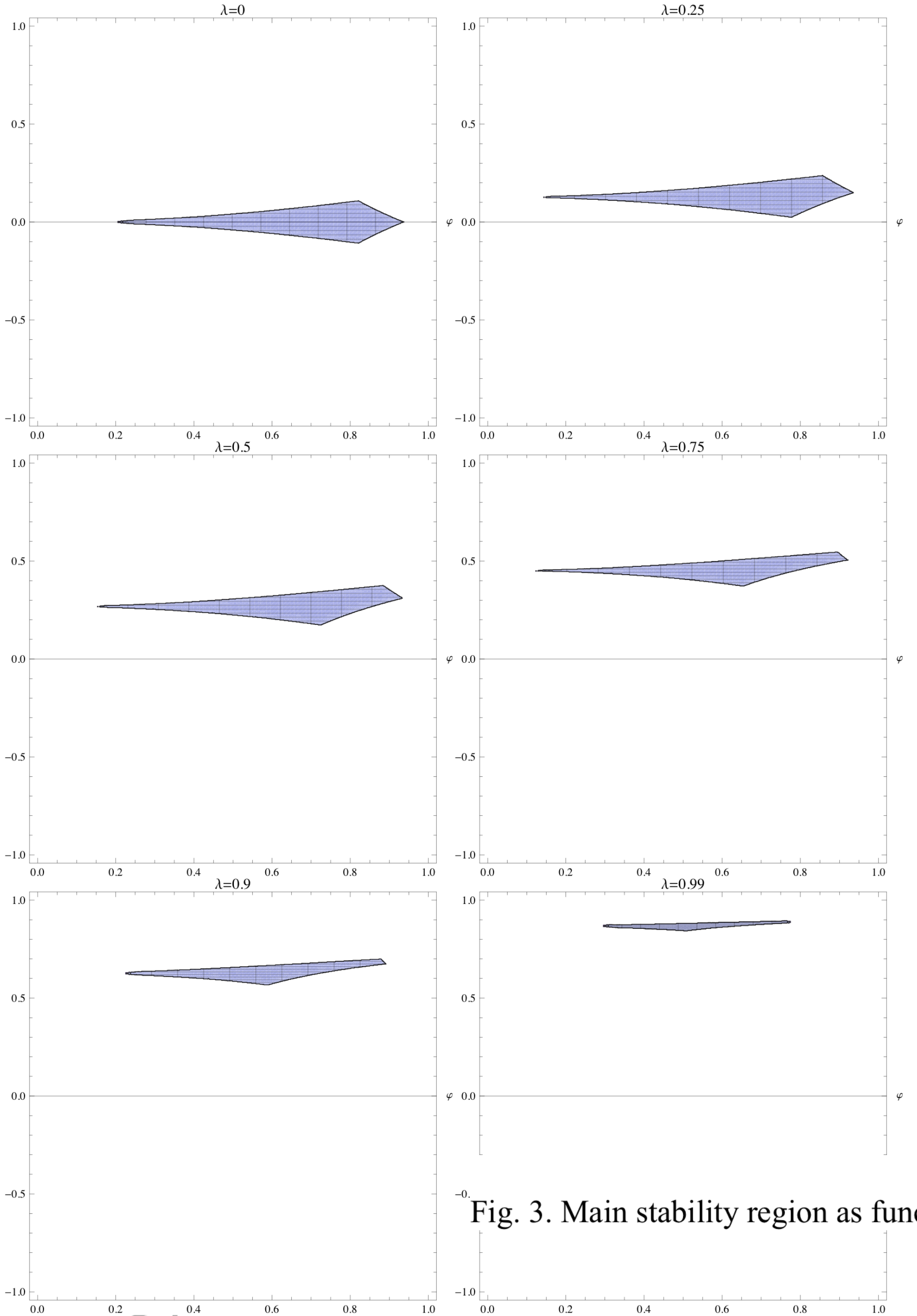
Phase

g=10



g=5



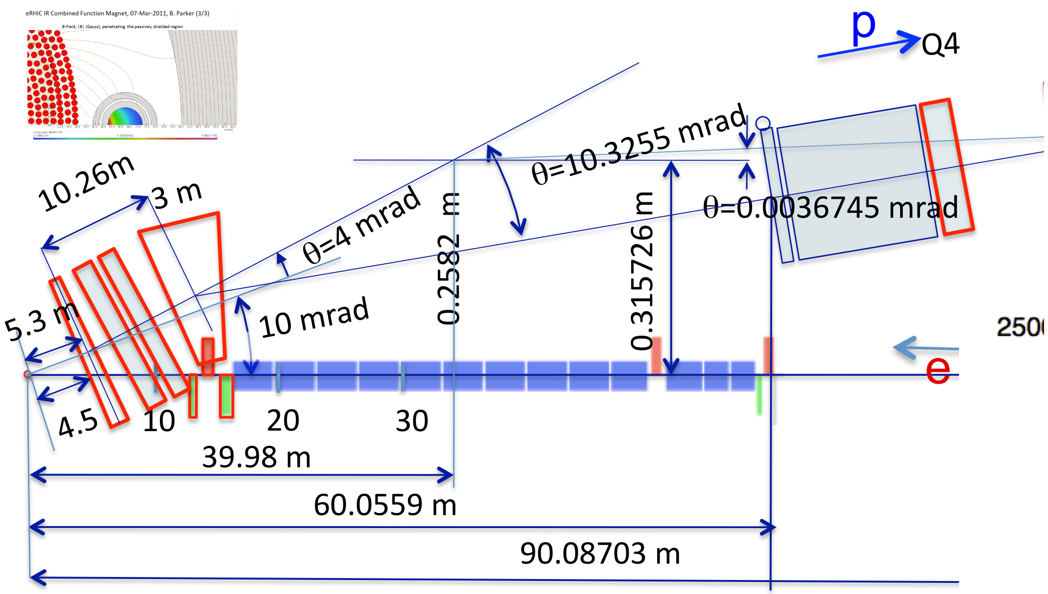


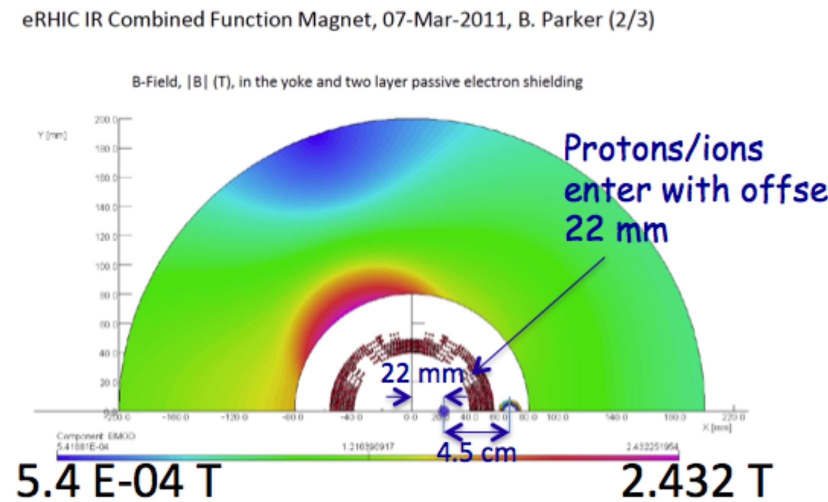
$$1 > \cos\mu_{x,y} = 1 + 2b_{x,yt}c_{x,yt} > -1; \Leftrightarrow 0 > b_{x,yt}c_{x,yt} > -1$$

$$\beta_{x,yF} = \sqrt{-\frac{d_{x,yt}b_{x,yt}}{a_{x,yt}c_{x,yt}}}; \beta_{x,yD} = \sqrt{-\frac{a_{x,yt}b_{x,yt}}{d_{x,yt}c_{x,yt}}}.$$

Fig. 3. Main stability region as function of λ. Vertical axis is ε.

eRHIC high-luminosity IR with β*=5 cm





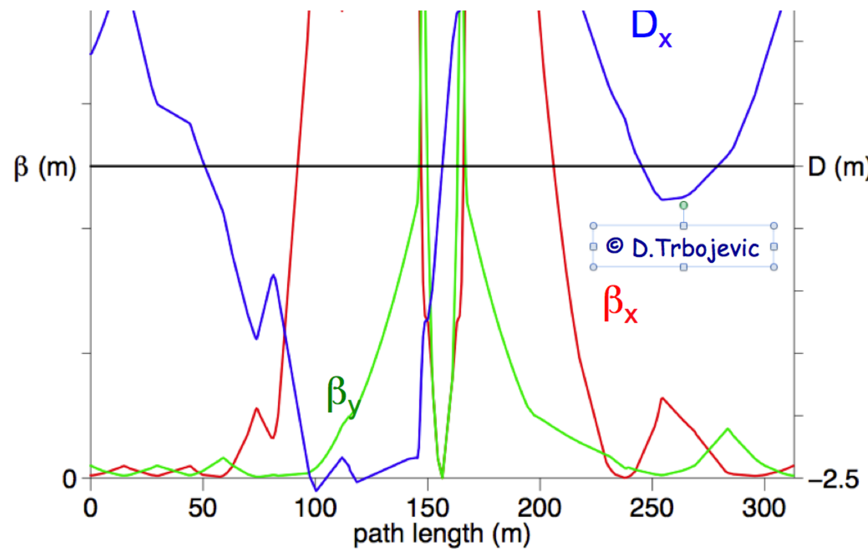


Fig. 1 Crab crossing scheme for KEKB

- 10 mrad crossing angle and crab-cros
- High gradient (200 T/m) large apertu
- Arranged free-field electron pass thi
- Integration with the detector: effici low angle collision products
- Gentle bending of the electrons to a

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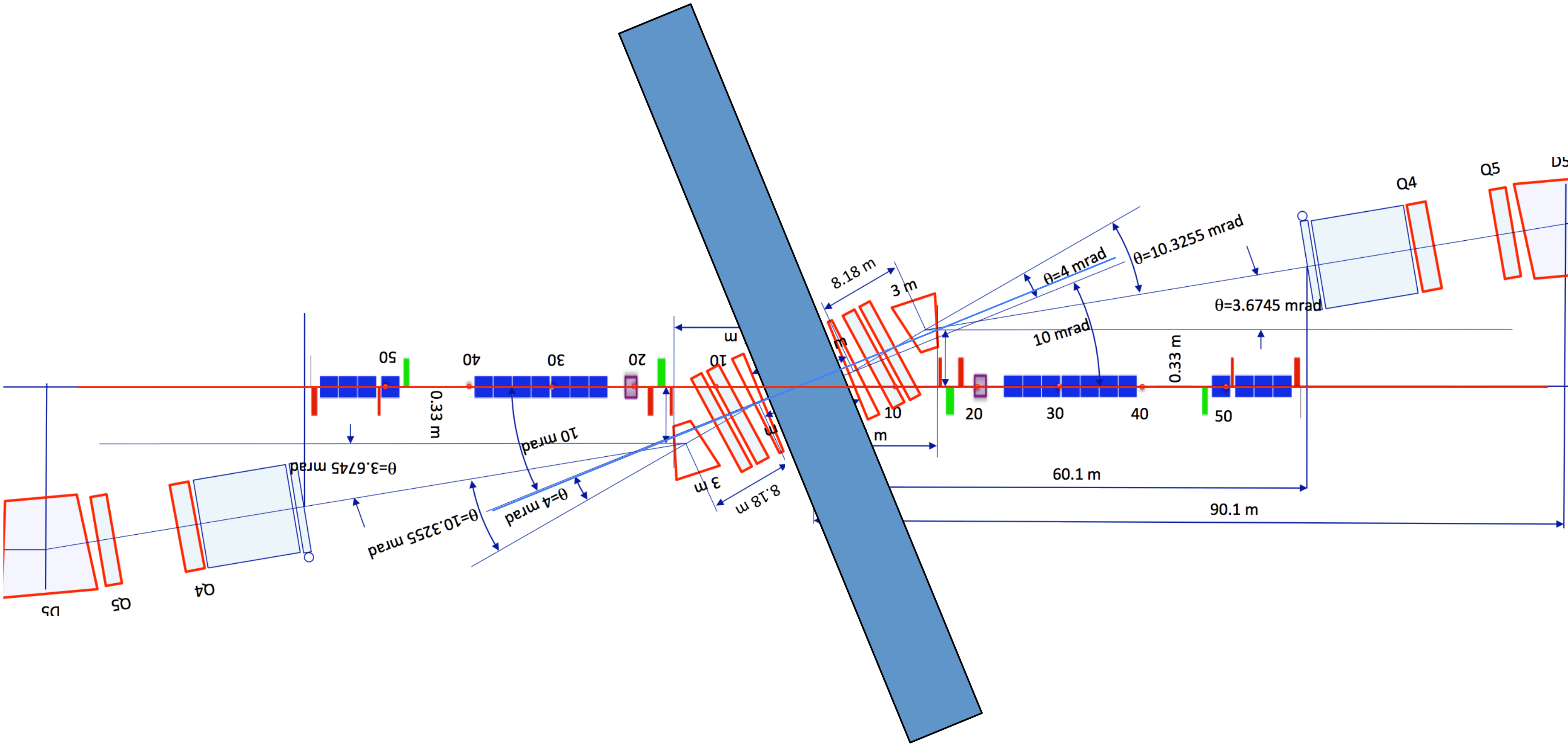
IR layout

- Vertical scale is distorted: magnification 40X



- Full scale looks like this:

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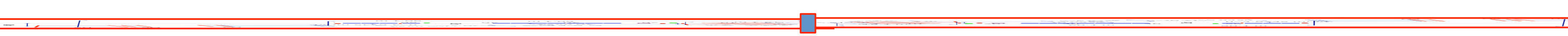
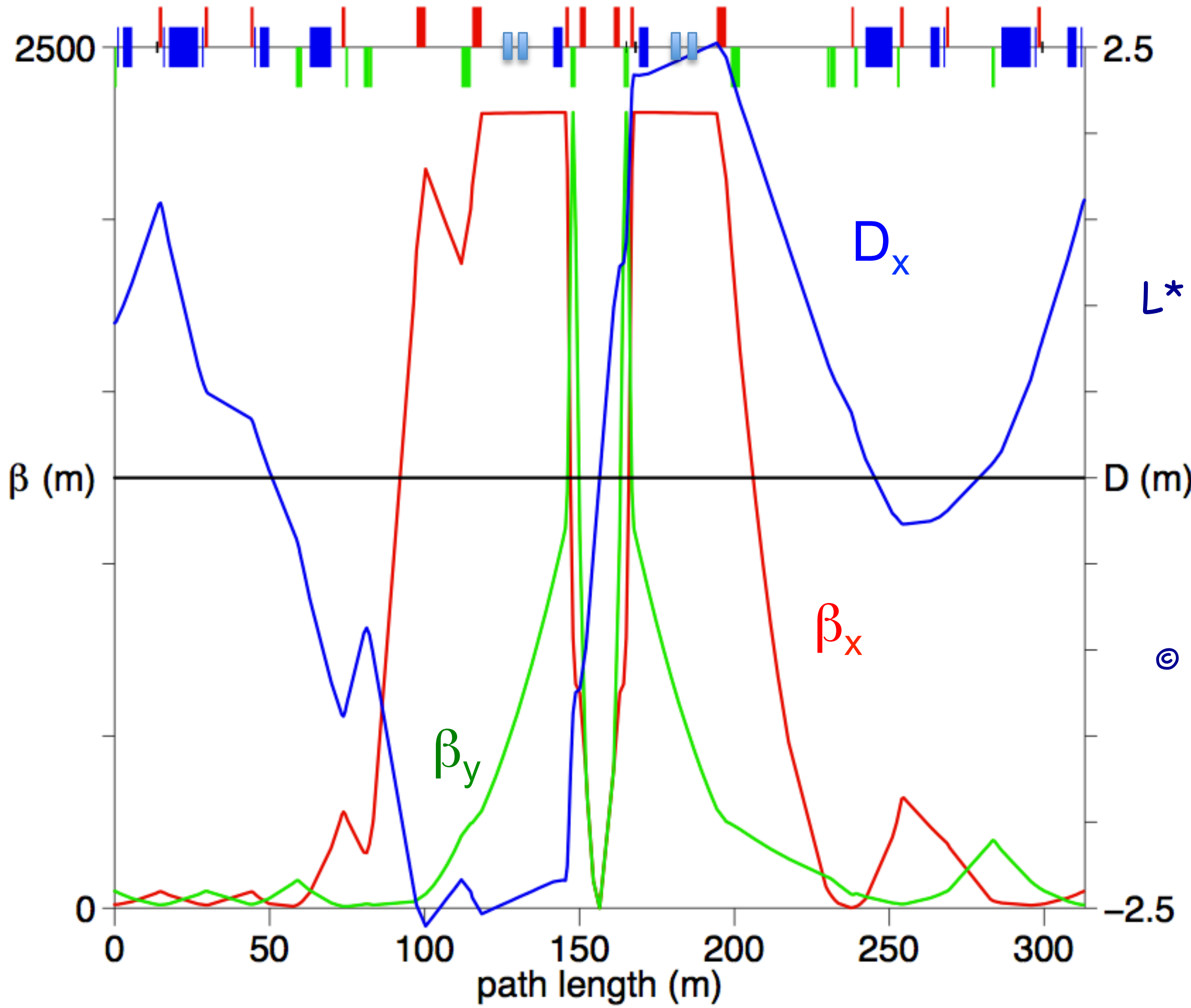


RHIC interaction region with β*= 5 cm & crab cavities Hadron lattice



$L^*_{triplet}$=4.5 m

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Major modification to RHIC

Modification to IR straights
Coherent Electron Cooling
6MV, 704 MHz RF system
Copper layered beam pipe
Crab cavities
Upgraded injection kickers (for 166 bunch operation)
Various Feedbacks, including that for kink instability suppression

BROOKHAVEN NATIONAL LABORATORY

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

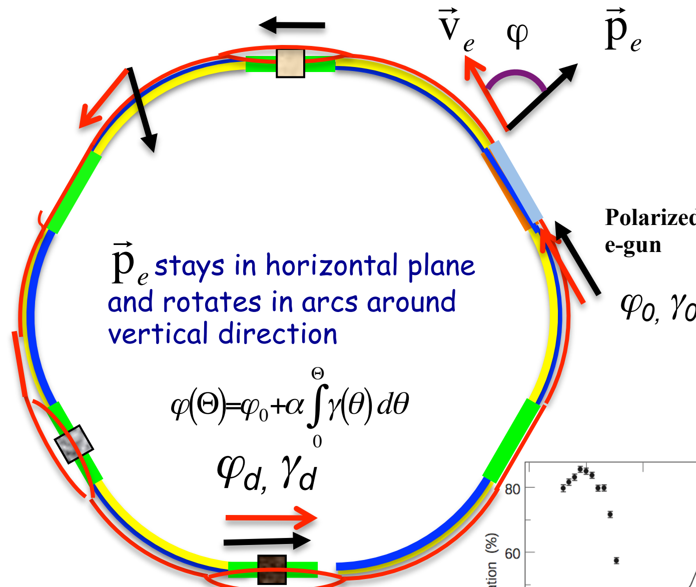
Main Accelerator Challenges for eRHIC

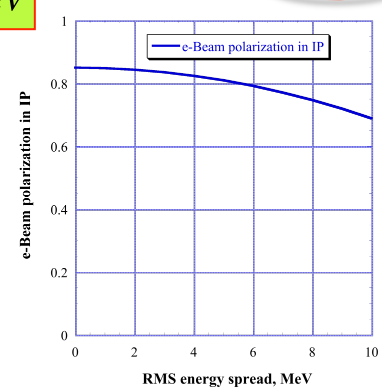
In red –increase/reduction beyond the state of the art

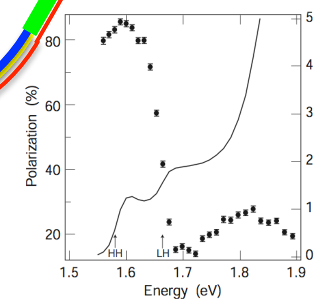
Polarized electron gun - 10x increase
Coherent Electron Cooling – New concept
Multi-pass SRF ERL 5x increase in current 30x increase in energy 3 x in # of passes
Crab crossing New for hadrons
Polarized ^{3}He production
Understanding of beam-beam affects New type of collider
β^*=5 cm 5x reduction
Feedback for kink instability suppression Novel concept

Electron polarization in eRHIC

- Only longitudinal polarization is needed in the IPs
- High quality longitudinally polarized e-beam will be generated by DC guns with strained-layer super-lattice GaAs-photocathode
- Direction of polarization will be switch by changing helicity of laser photons in and arbitrary bunch-by-bunch pattern
- We continue relying on our original idea *(©VL 2003)* to rotate spin integer number of 180-degrees between the gun and the IP
- With six passes in ERL the required condition will be satisfied at electron energies: $E_e = N \cdot 0.07216\,GeV$
- It means that tuning energy in steps of 72 MeV (0.24% of the top energy of 30 GeV) will provide for such condition
- Energy spread of electrons should kept below 6 MeV to have e-beam polarization in IP above 80%



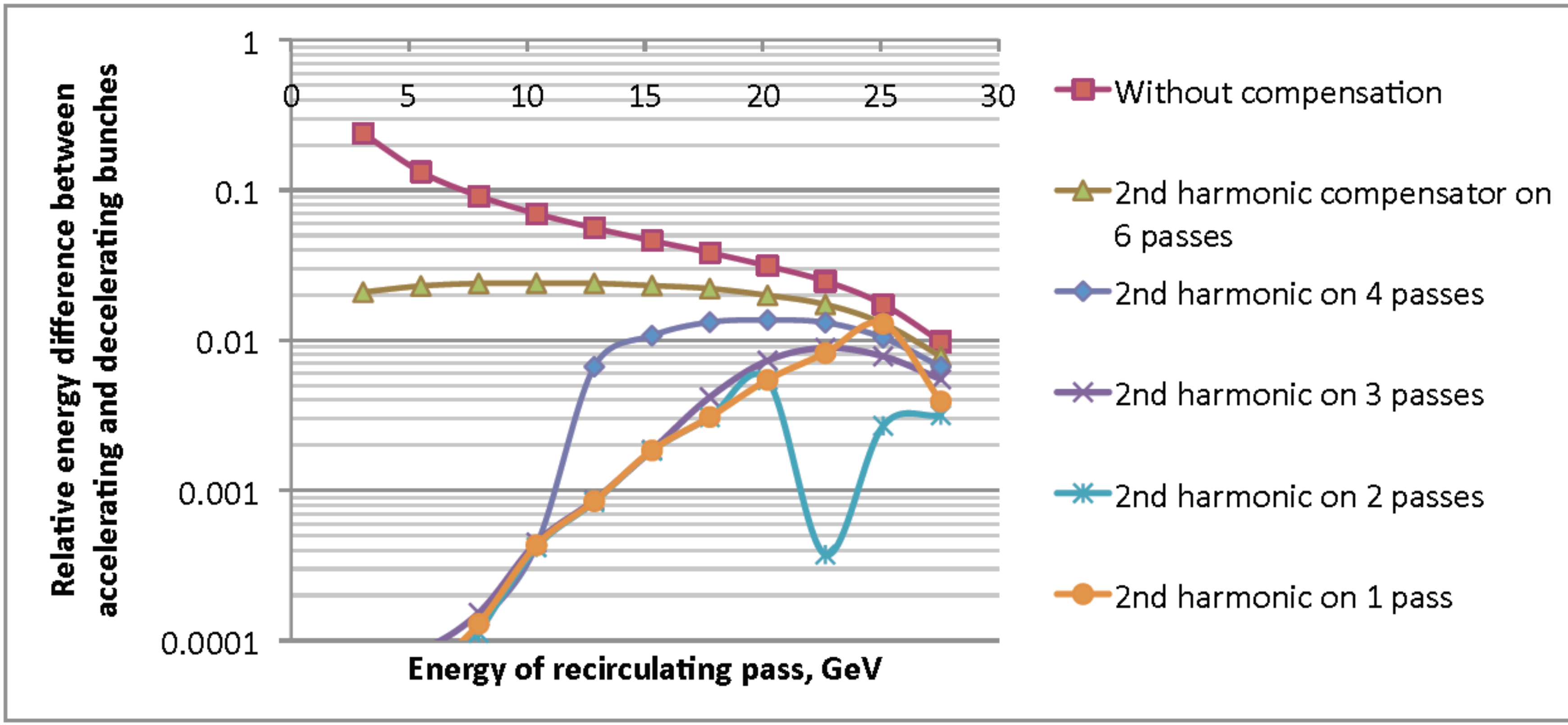




***The GaAs–GaAsP cathode achieved a maximum polarization of 92±6% with a quantum efficiency of 0.5%**

Highly polarized electrons from ..strained-layer super-lattice photocathodes, T. Nishitani et al., J. OF APPL. PHYSICS 97, 094907 (2005)

Loss budget for 6 pass scheme



Details in talk by V.Ptitsyn

Coherent Electron Cooling vs. IBS: 250 GeV, $N_p=2\cdot 10^{11}$

$$X=\frac{\tau_{CeC}}{\sqrt{\tau_{IBS//}\tau_{IBS\perp}}}\frac{1}{\sqrt{\xi_\perp(1-2\xi_\perp)}};\quad S=\frac{\tau_{CeC}}{\tau_{IBS//}}\cdot\sqrt{\frac{\tau_{IBS\perp}}{\tau_{IBS//}}}\cdot\sqrt{\frac{\xi_\perp}{(1-2\xi_\perp)^3}}$$

$$X=\frac{\varepsilon_x}{\varepsilon_{xo}};\ S=\left(\frac{\sigma_s}{\sigma_{so}}\right)^2=\left(\frac{\sigma_E}{\sigma_{sE}}\right)^2;$$

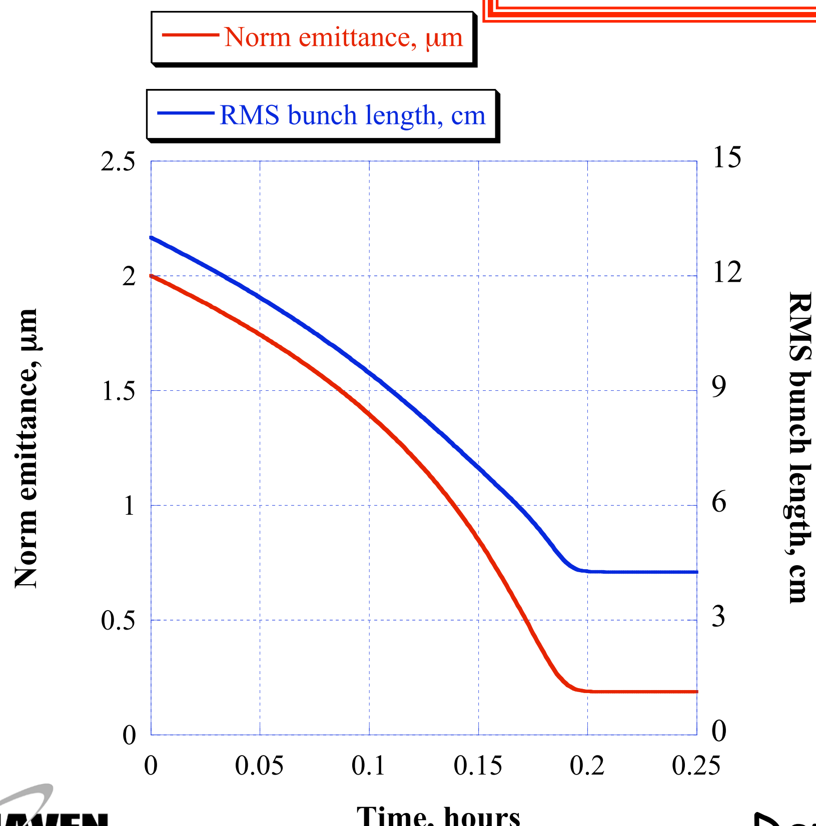
$$\frac{dX}{dt}=\frac{1}{\tau_{IBS\perp}}\frac{1}{X^{3/2}S^{1/2}}-\frac{\xi_\perp}{\tau_{CeC}}\frac{1}{S};$$

$$\frac{dS}{dt}=\frac{1}{\tau_{IBS//}}\frac{1}{X^{3/2}Y}-\frac{1-2\xi_\perp}{\tau_{CeC}}\frac{1}{X};$$

$$\varepsilon_{xn0}=2\,\mu m;\ \sigma_{s0}=13\ cm;\ \sigma_{\delta 0}=4\cdot 10^{-4}$$
$$\tau_{IBS\perp}=4.6\ \ hrs;\ \tau_{IBS//}=1.6\ \ hrs$$

IBS growth time calculated by by A.Fedotov using Beta-cool

$$\varepsilon_{xn}=0.2\,\mu m;\ \sigma_s=4.9\ \ \text{cm}$$



Important for:

a) keeping the luminosity constant
b) Reducing need for polarized beam current
c) Increase electron beam energy to 30 GeV
d) Increase luminosity by reducing β* to 5 cm

Details in talk by V.Litvinenko